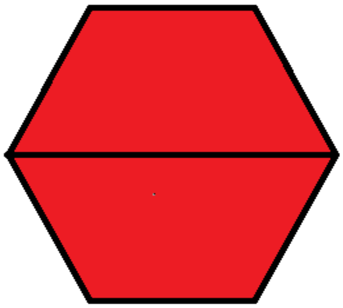
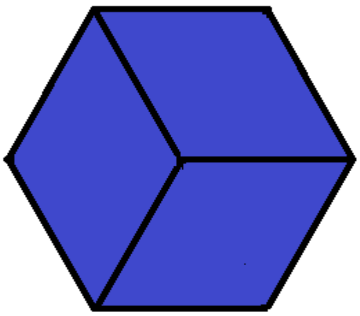
Hexagon Fractions

A hexagon can be divided up into several equal, different-sized parts. For example, if we draw a line across the center, like this:

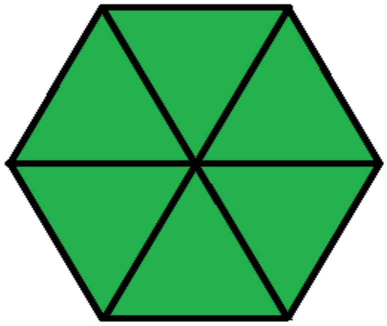


We have separated the hexagon into **two** equal-sized parts. Each one is **one-half** of the hexagon. We could draw the lines differently:



Here we have separated the hexagon into **three** equal-sized parts. Each one of these is **one-third** of the hexagon.

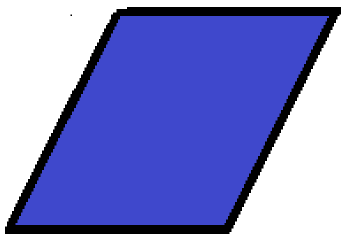
Another way of separating the hexagon into equal-sized parts looks like this:



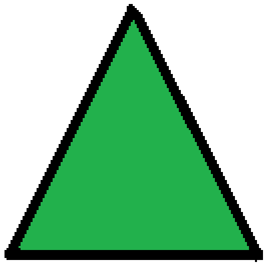
There are **six** equal-sized parts here, so each one is **one-sixth** of the hexagon.

Each of these shapes has a special name. The shape that is **one-half** of the hexagon is called a **trapezoid**:

The one that is **one-third** of the hexagon is a **rhombus**:



and the one that is **one-sixth** of the hexagon is a …



Yes, a **triangle**! You knew that one, for sure.

We can use these shapes to get an understanding of how fractions are added and subtracted. Remember that the numerator of a fraction is just a regular number. The denominator of a fraction is more than just a number, it is sort of a "thing," like apple, dog, or flower. But it is a "numerical thing" and has a *name*. We should think of "half" or "third" or "tenth" or whatever as a *name*, just like "apple" or "dog" or "flower" is the name of something. The numerator in a fraction tells us how many of the denominators we have.

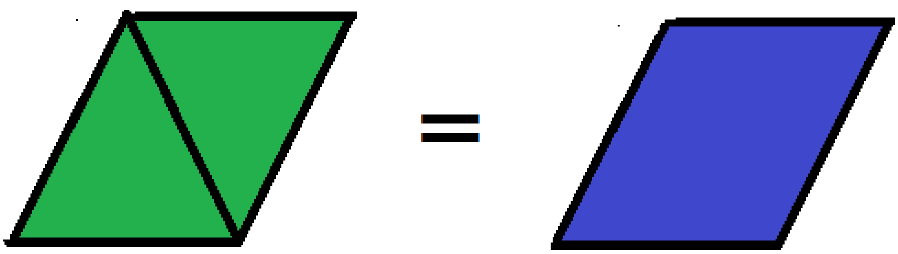
For example, because the rhombus is one-third of a hexagon, we can think of the fraction "one-third" as "one rhombus." For a hexagon, "third" and "rhombus" are two names for the same thing. The fraction "two-thirds" would be "two rhombuses." The numerator tells us how many of the denominators we have, and *when we are looking at a hexagon*, the denominator "third" means "rhombus." It's a *thing* as well as a number.

The fraction "four-sixths" would mean "four triangles," because one-sixth of the hexagon is a triangle.

In Book 1 we learned how to add and subtract "like fractions." You learned that like fractions have the same denominator, and when you add them, the denominator of the answer has to stay the same. We said that different denominators would be like puppies and kittens; you can't add two puppies and three kittens together to get five *anythings*! In the same way, you can't just add numerators when the denominators are *different*. **Thirds and fifths are as different as puppies and kittens.** You know that when the denominators are the same, you just have to add the numerators, and the denominator can't change. Things are a little different when the denominators are not the same.

Before explaining how to add and subtract unlike fractions, that is, fractions with different denominators, we're going to look at a non-mathematical way of adding and subtracting them. What we're going to look at works only for hexagon fractions, but it is very useful for understanding the rules for all unlike fractions.

Let's start by looking at how these three different shapes are related to one another. If we put two triangles next to each other, you can see they form a rhombus:



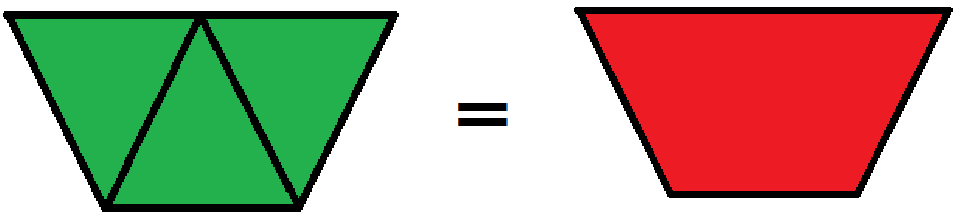
We could say that "one triangle plus one triangle equals one rhombus" or we could say that "one-sixth plus one-sixth equals one-third," because one triangle is one-sixth and one rhombus is one-third of the hexagon. These are two different ways of saying the same thing. The first one uses the "shape-thing" as the denominator, and the second one uses the "number-thing" as the denominator. We can write the mathematical equation like this:

$$\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

A different way of describing what we have done by placing two triangles together is to say "Two triangles equal one rhombus." This time we are multiplying one triangle by the number, two, to get a rhombus. Using fractions instead of shapes, we would say, "Two times one-sixth equals one-third," and write it like this:

$$2 \text{ x } \frac{1}{6} = \frac{1}{3}$$

If we put three triangles next to each other, you can see they form a trapezoid:



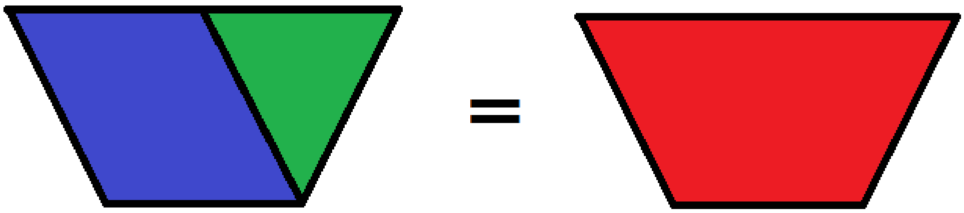
In this case we can say, "one-sixth plus one-sixth plus one-sixth equals one-half" (remember that a trapezoid is one-half of a hexagon), or we can say "three times one-sixth equals one-half." These equations can be written:

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

and

$$3 \text{ x } \frac{1}{6} = \frac{1}{2}$$

If you examine the picture more closely, you can see that the trapezoid could be the sum of a rhombus and a triangle:



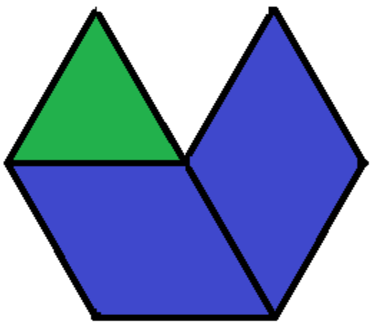
The mathematical equation for this is:

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

We can use these shapes to do more complex additions. For example, suppose we want to add two-thirds and one-sixth:

$$\frac{2}{3} + \frac{1}{6}$$

We can arrange two rhombuses (each rhombus is one-third) and one triangle like this:



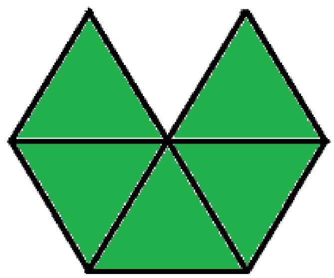
Since the rhombus is a third and a triangle is a sixth, we can write their sum as:

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{6}$$

Which is the same as:

$$\frac{2}{3} + \frac{1}{6}$$

Now all you have to do is remember that a rhombus is equal to two triangles, and you know that the figure above must be the same as five triangles; two for each rhombus and the one triangle in the figure:



But “five triangles” is the same as “five sixths” for a hexagon, so the answer to our equation must be:

$$\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

Another way of looking at the figure above is to see that it is one triangle less than a full hexagon. We can write this in a mathematical equation like this:

$$\frac{6}{6} - \frac{1}{6} = \frac{5}{6}$$

Where you have to remember that one whole hexagon can be written as six-sixths.

In fact, you can see that this is true by looking at the first three pictures in this handout. The first whole hexagon is two-halves, the second whole hexagon is three-thirds, and the third whole hexagon is six-sixths.

Using hexagons to help us visualize fractions is limited to a very small number of fractions, but it can be very helpful in understanding the fundamental nature of *all* fractions.

Dividing Fractions

Using the relationships among hexagon fractions can be very helpful in understanding what it means to divide by a fraction. Let's start by reviewing what "division" itself means.

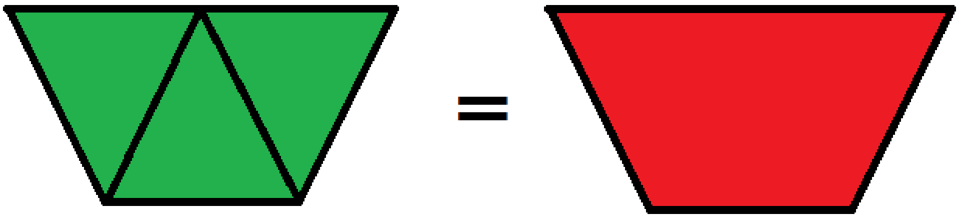
When we divide ten by two, for example, we do so to find out how many twos are in ten, and the answer is five. We can write this as a mathematical equation in two common ways:

$$10 \div 2 = 5 \quad \text{or} \quad 10/2 = 5$$

Either equation tells us there are five twos in ten. In general, when we divide one number by another, the answer tells us how many of the bottom number fit into the top number.

What if the numbers are fractions? For example, "What is one-half divided by one-sixth?" You might not think you know how to do this division problem, but it is really very simple if you first realize that the question is the same as, "How many one-sixths are there in one-half?" *and you recall the hexagon fractions we've talked about.*

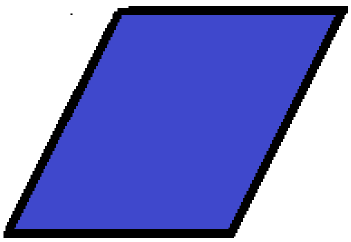
Remember that one-sixth of a hexagon is a triangle and one-half of a hexagon is a trapezoid. And that the trapezoid can be made up of three triangles:



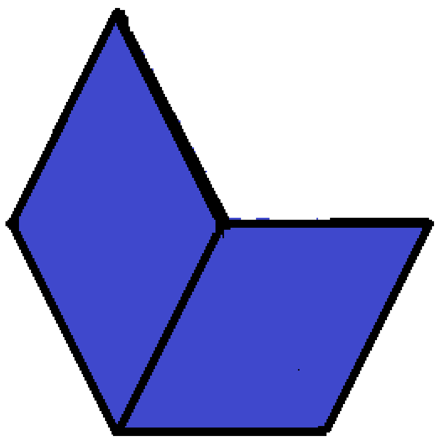
You don't have to know any math to see that the number of triangles in the trapezoid is three. But the fact is, the trapezoid is one-half and the triangle is one-sixth (of a hexagon), so there are three one-sixths in one-half. Or to put it another way, one-half divided by one-sixth equals three.

What is two-thirds divided by one-sixth? In other words, how many one-sixths fit into two-thirds?

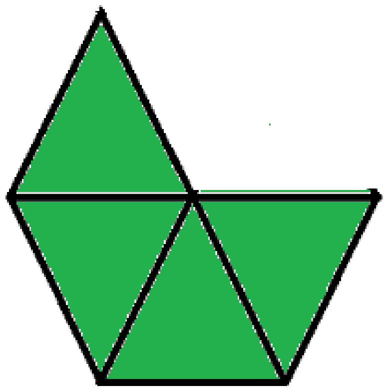
The answer is obvious when we look at hexagon fractions: One-third is a rhombus:



And two-thirds is two rhombuses:



On page 4 we saw that one rhombus is the same as two triangles, so we know that two rhombuses is the same as four triangles:



It is very easy to see that there are four triangles here. But remember that a triangle is the same as one-sixth, and two rhombuses is the same thing as two-thirds. We have just figured out that two-thirds divided by one-sixth is just four!

In fact there is a simple rule that says when you divide by a fraction, that's exactly the same thing as inverting the fraction first, then multiplying by it. For example, let's divide one by one-half. The rule says invert the fraction, then multiply. One-half inverted is just two, so we multiply one by two and the answer is two: There are two one-halves in one.

Now let's go back and look at the two problems we just did using hexagon parts and no math.

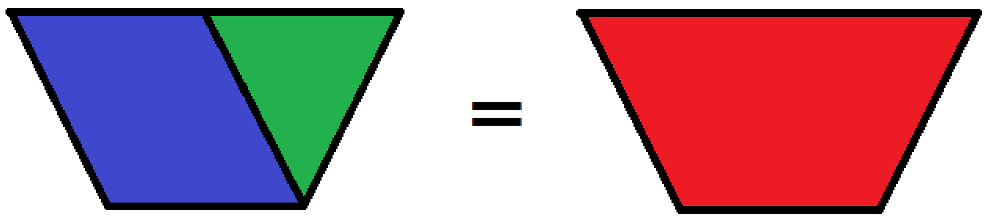
The first one was one-half divided by one-sixth. Invert the one-sixth to get six, then multiply one-half by six. One-half times six is just three, and that's the same answer we got by just looking at hexagon parts.

The second one was two-thirds divided by one-sixth. Again we invert the one-sixth to get six, and this time we multiply two-thirds by six. One-third of six is two, so two-thirds of six is four, the same answer we got using hexagon parts.

Remember that we invert only the fraction in the denominator; if there's a fraction in the numerator, we leave that as it is.

Let's do one more example. This one is a little more complicated, but you can understand it:

What is one-half divided by one-third? In other words, how many one-thirds fit into one-half? On page 6 there as a picture of one rhombus and one triangle on one side of an equation, and a trapezoid on the other:



We used this picture to show that one-third plus one-sixth equals one-half.

We can look at this same picture in another way: "The rhombus and the triangle together equal a trapezoid, and we want to know how many rhombuses fit into a trapezoid." Using the appropriate fractions, this is the same thing as saying "One-third and one-sixth together equal one half, and we want to know how many one-thirds fit into one-half."

The way to solve this is to recognize that the triangle is exactly one-half of the rhombus. So the rhombus is ONE rhombus, and the triangle is ONE-HALF of a rhombus. Together they equal one trapezoid. That is the same thing as saying, "The trapezoid is one and one-half rhombuses," OR, "There are one and one-half thirds in one-half." Written mathematically,

$$\frac{1}{2} \div \frac{1}{3} = 1\frac{1}{2}$$

Finally, let's use our "invert and multiply" rule to solve this division problem: Invert the one-third to get three over one, then multiply by one-half:

$$\frac{1}{2} \text{ x } \frac{3}{1} = \frac{3}{2} = 1\frac{1}{2}$$

One-half of three is of course one and one-half, and again, this is the same answer we got by looking at hexagon parts.