Learning Math with Kayla

Book 7: Dividing fractions

Vicki Meyer

Illustrator Sue Lynn Cotton

The Learning Math with Kayla Books

Book 1 Adding and subtracting like fractions
Book 2 Multiplying fractions
Book 3 Learning multiplication facts
Book 4 Place values, Multiplying large numbers
Book 5 Adding and subtracting unlike fractions
Book 6 Learning about improper fractions and mixed numbers
Book 7 Dividing fractions
Book 8 Adding and subtracting large numbers
Book 9 Solving long division problems
Book 10 Working with decimals and percents
Book 11 Learning about negative numbers
Book 12 Problem solving!

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About the Kayla Books

The Kayla books tell the story of a fourth grade girl who has gotten so far behind in her math class that she is not able to understand what her teacher is trying to teach her. Her math teacher, Mr. Williams, is aware of how poorly Kayla is doing. He decides a tutor would be the best way to help Kayla learn her math.

In this seventh book Kayla successfully learns how to divide by a fraction. She is eager to show her mother how the "invert and multiply" rule works. Kayla is becoming more confident in her ability to learn math. In fact, she is making math one of her priorities.

There are twelve books in this series. Whether you’re a fourth grader, in middle school or in high school; a Mom or Dad or a Grandparent, you can learn along with Kayla.

The story is told by Kayla, right before she goes off to college.

About Kayla

I have been asked if Kayla is a real person. She and others in the Kayla books are composites of the many kids I have tutored plus myself as a kid *and* as an adult. Like Kayla, I remember how nervous I was making my first telephone call. I wanted to do everything right but my throat tightened up and I couldn't say anything.

It is very common for kids, after they do the math, to forget what the original question was. Kayla did that on page 23. Ms. Gibbs asked her “How much pizza does each student get?” and Kayla just answered “Huh?”   I remember forgetting the original question, too, as do many of the kids I tutor.

About the Author

After Vicki raised six really smart kids, she began studying for her Ph.D. in order to keep up with them. She taught at the university level for 25 years, then began tutoring elementary school students. Vicki soon found a new career for herself, tutoring math for at-risk kids, writing about her experiences, and putting together the Kayla books.

Anne Rainey, with Vicki on the back cover, teaches college students who are studying to be math teachers. She made significant contributions to the writing of this book. Anne lives with her husband, Vicki's son, John, in Michigan.

Acknowledgements

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Thanks to Charles Daniel for the excellent job he does in proofreading the text. His expertise in mathematics and grammar ensures that both kinds of errors are minimized in the Kayla Books.

And a special thanks to my husband, Ed, for all of his great suggestions, his skillful editing, and especially his patience. I would not be able to complete the books without him.

DEDICATION

To my mother, Phyllis Hurtova, who was prevented from going past the fourth grade due to political unrest in Czechoslovakia, but continued to be a life-long learner.

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Chapter 1

I’m invited to a pizza party!

It’s Friday. We have only a half-day of school today. That’s because the teachers are having a meeting or something. And guess what? Cleveland asked me if I wanted to have pizza at his house tonight!

Well he didn’t *really* ask me, his little brother did. His name is Jared and he’s in second grade. Sometimes we play in the playground together. He’s teaching me how to dribble.

Dribbling is just bouncing a basketball but you don’t use your whole hand, you just use your fingertips. Got it?

Cleveland taught Jared how to dribble and now Jared is teaching me. They’re both real good. Cleveland is the best, of course, Jared is second, and well…I’m not so good but I’m getting better. That’s because I practice.

You have to practice if you want to be a good dribbler. Actually, you have to practice if you want to be good at *anything*, even math, ‘cause if you don’t, you just won’t be very good.

Well anyway, when Jared asked me if I wanted to come over for pizza, I looked over at Cleveland and he said, “Sure, why not?”

I love pizza but we almost never have it. I have to ask my momma if I can go to Cleveland’s house for pizza. She only lets me go to someone’s house if she knows the family. She has gotten to know Cleveland’s family and so I’m pretty sure she’ll let me go.

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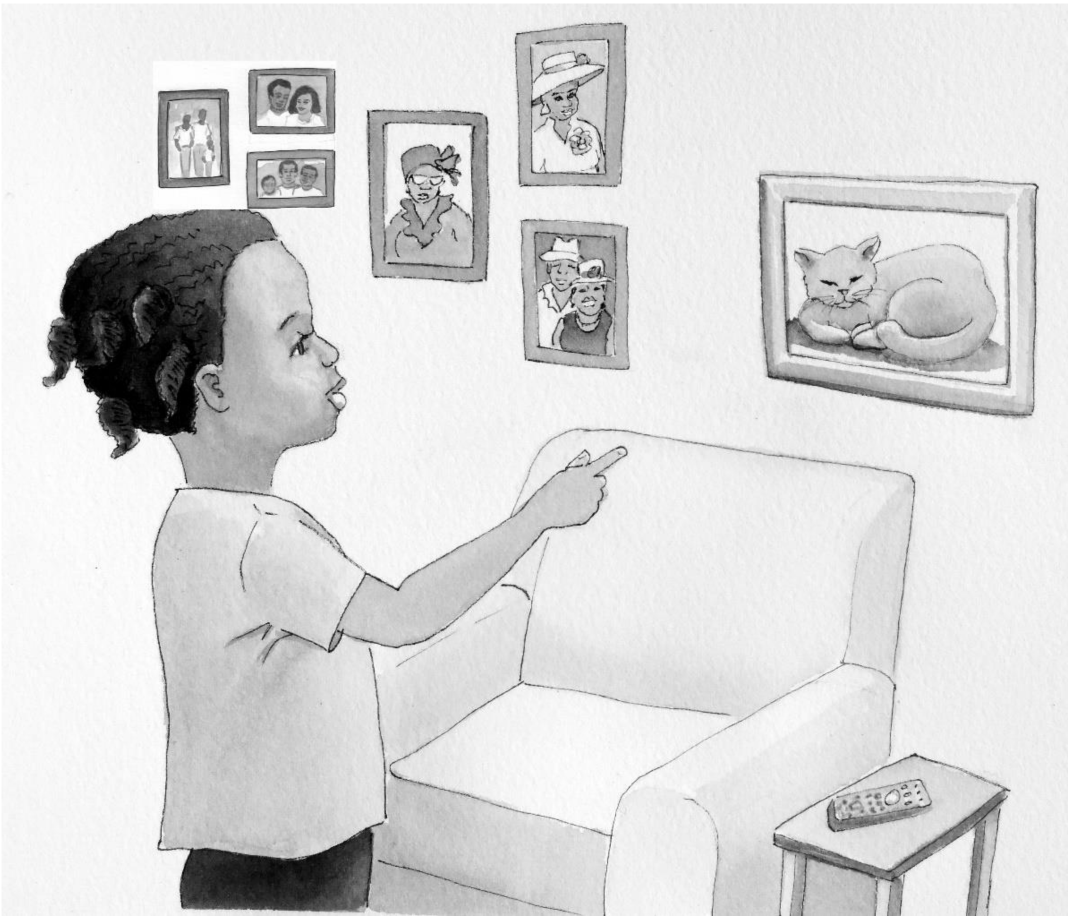
Well, to make a long story short, I asked her, she said, “Yes,” and so I went.

The very first thing I noticed when I went into Cleveland’s house was the pictures on the walls. There were lots of them. Most of them were family photos but there was one picture that wasn’t. It was a drawing of a cat.



I don’t think it’s Cleveland’s cat, though.   It was different. It must have come from the store because it was in a frame. And besides, it was really, really, good.

Cleveland introduced me to his family: his father, his big brother, TJ, and his aunt, Tycena. Jared was there too, but Cleveland didn’t introduce me to him because I already knew him.

Cleveland’s father told Cleveland he could call the pizza store and order the pizza. “But first,” he said, “find out what kind of pizza everyone wants and how much they want. Your mother always takes two pieces of pepperoni pizza. I’ll take three pieces, pepperoni or cheese, doesn’t matter much, I like them both.”

Chapter 2

Taking pizza orders

“Cleveland, could I take the orders for the pizza, pleeeeze?” I asked. He just said, “Sure.”

“Oh, thank you, and oh, I need to know how many pieces there are in each pizza. That’s important because I have to know what to put for the denominator.”

“The denominator?” Cleveland asked surprised. “Why would you need a denominator?”

“Well you see, pizzas are just like fraction bars but instead of coloring the sections, you get to eat them. The number of pieces in each pizza is what the denominator is,” I patiently explained.

“We always get large pizzas and there are eight pieces in each one. Oh, and there’s a pad on the table in the front hall,” Cleveland offered. “You can use it to write stuff down.”

OK, so now I have a pad to take everyone’s pizza order and I know the denominator is eight so I’m all set to take everyone’s order. I was so excited! I wanted to do everything right.

First I made two columns and put a “C” and a “P” on top of each column. These letters stand for cheese and pepperoni. This is going to be fun!

I already knew what Cleveland’s parents wanted and I knew that each pizza was divided into eight pieces so I wrote “**3/8**” in the “C” column for Cleveland’s father and “**2/8**” in the “P” column for his mother

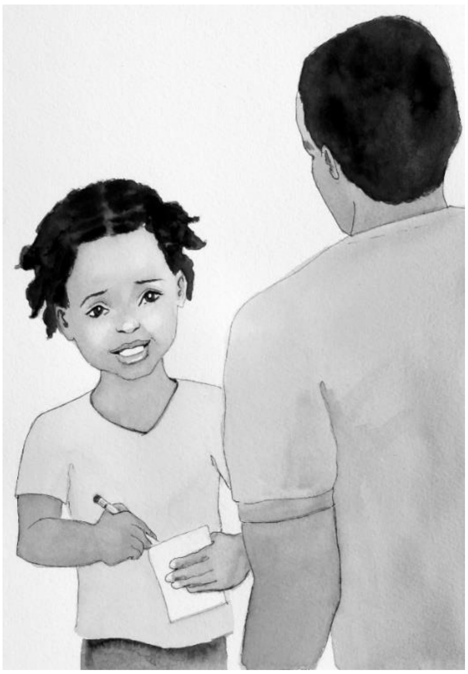
Now I remembered that Cleveland’s father said it didn’t matter what kind of pizza he ate, but I *had* to put the three-eighths down somewhere, now didn’t I?

OK, that’s done, now I’m ready to take everyone else’s order. Who should I ask first? I could see TJ was in the next room so that’s who I asked first.

TJ doesn’t have a name, he just has initials. But that’s O.K. I just call him TJ.

I was a little nervous but I tried hard to sound very grown-up like. I cleared my throat first and then said, “TJ, your father asked Cleveland to take orders for pizza and I’m helping him. What kind of pizza would you like and how much of it would you like?”

I held my pad up ready to write his order down.



All he said was, “Huh?” So I started again, “Your father asked us...” TJ interrupted me and said, “My father’s not here.”

Now I was the one who said, “Huh?” I looked over at Cleveland. I didn’t ask him anything but he answered me just the same. “Oh, he’s my half-brother.”

A half-brother! How could he be a half of a brother? I wondered.

“Hey, TJ,”, Cleveland called over, “I know you like pepperoni pizza, how many pieces do you want?”

“Four.” That’s all he said.

So I wrote down “**4/8**” after TJ’s name in the “P” column.

I better leave out the part about Cleveland’s father when I’m taking orders. So for everyone else in the house, I just said, “I’m taking pizza orders, can you please tell me what kind of pizza you would like and how much of it you want?” And everyone I asked told me what kind of pizza and how much of it they wanted.

OK, now I’m ready to add up the fractions.

“Don’t forget to order some pizza for Rakiah,” Cleveland said.

“Rakiah? Who’s Rakiah?” I asked.

“Rakiah is my sister,” he said. “Right now she’s out running but she’ll be home in time to take a shower and eat pizza with us.

She’s practicing for a track meet at the high school. She usually wins because she’s so fast. And she’s so fast because she practices a whole lot. You better put down three pieces for her. Oh, and she likes cheese.”

So I added Rakiah’s name and put a “**3/8**” in the “C” column after her name. I started to add up the fractions again.

Cleveland was looking over my shoulder at my list. He said, “Hey Kayla, I don’t see my name on that list. I’ll take three slices of pepperoni. And what about you? Don’t you want any pizza?”

“Oh!” I quickly put our names on the list and put a “**3/8**” in the “P” column for Cleveland and a “**2/8**” in the “C” column for me.

OK, I think I’m all done. All I have to do now is add up the fractions.

Pizza	C	P
C’s Dad	3/8	
C’s Mom		2/8
T J		4/8
Jared	2/8	
Tyeena	3/8	
Rakiah	3/8	
Cleveland		3/8
Kayla	2/8	

Reader, add the fractions and compare your answers with Kayla's. Imagine yourself ordering the pizza. Think about what you would tell the pizza man.

The denominators were all the same so it was easy to add up the fractions. I did it twice because I didn't want to make any mistakes.

I'm pretty sure they're right. But the answers look kinda funny:

Cheese **13/8** Pepperoni **9/8**

Oh, both fractions are improper. I better change them into mixed numbers:

Cheese $\mathbf{1\frac{5}{8}}$ Pepperoni $\mathbf{1\frac{1}{8}}$

There, now they look better.

Then I remembered about simplifying. I tried to make the numbers smaller but I just couldn't. But it's not my fault! They just weren't the right kind of numbers for doing that!

I told Cleveland I was all done taking orders for the pizza.

Cleveland said, "Good. I'm getting hungry! I'll call the pizza place."

"Oh, can I call the pizza place?" I asked Cleveland, "Pleeeeze?"

"Sure," he said as he punched in the numbers on his cell phone. I quickly practiced what I was going to order - one and five-eighths of cheese pizza and one and one-eighth of pepperoni pizza.

Cleveland handed me his phone and said “Make sure you give them our address. Here, I’ll write it down for you.”

I took the phone from Cleveland and listened. A man answered. I didn’t understand all he said but I did hear the word “pizza” in there somewhere so it must be the right place.

I tried to sound grown-up, “Hello. I’d like to order some pizza. I would like one and five-eighths of a cheese pizza and one and one-eighth of a pepperoni. Here is the addre....”

The man on the phone interrupted me. He said, kinda gruff-like, “Listen, kid. We only sell whole pizzas, not eighths. I don’t have time for this. Call back when you got the order right, OK?” And then he just hung up!

I didn’t know what to do.

Cleveland was standing nearby. I think he figured out something was wrong. "I'll just have my Dad call. He usually does anyway," and then he added, "Hey, Kayla, don't worry about it."

But I was worried about it.

Cleveland's Dad took the list from me, glanced at it, and then made the call. "I'd like two large cheese pizzas and one large pepperoni pizza. He gave the pizza man his address, thanked him, and then hung up.

I waited to see what would happen.

A little while later, the doorbell rang. It was the delivery man. He handed Cleveland's father three boxes of pizzas. At the very same time, Cleveland's mother walked in the door. And then I heard Rakiah running down the stairs and shouting, "Hey, I smell pizza, let's eat!"

And so we did. And guess what? All the pizza was eaten and nobody seemed to want any more or even want a different kind of pizza.

Cleveland’s father must have ordered the exact amount of pizza and the right kind! Well, he used my order sheet so I must have gotten *something* right!

As I was leaving the house, I stopped again to look at the pictures on the wall.

Cleveland’s father came over and said proudly, “Cleveland drew that picture of the cat.”

“Really?! But - but it’s in a frame. Doesn’t that mean you bought it at a store?” I asked.

“Well, I bought the frame from a store,” he said, “but Cleveland drew the picture. It’s quite good isn’t it?”

“It’s really, *really* good,” I answered, “I wish I could draw a cat like that.”

I thanked Cleveland’s father for letting me come over…looked at that cat one more time…and then I left.

Chapter 3

The diagonal

Today is tutoring day and I'm prepared for Ms. Gibbs. I've been studying my math for about fifteen minutes every day - even on Sunday - and I'm getting pretty good at working problems. You see, math is one of my priorities!

When I walked into the tutor room, Ms. Gibbs was already there. And she already had my multiplication grid on the table in front of where I sit. After we greeted each other, Ms. Gibbs asked me if I would be able to put the rest of the squared numbers in my grid today.

"Yes," I answered, "I studied my squared numbers and I know them well. Six squared is thirty-six, seven squared is forty-nine, and eight squared is sixty-four."

$$6^2 = 36$$
$$7^2 = 49$$
$$8^2 = 64$$

Ms. Gibbs handed me the grid and I put those squared numbers right where they belong. I wanted to count the number of boxes I had left to fill out - I could see that there weren't many. But right away, Ms. Gibbs took the grid from me.

She didn't put in back in my folder like she usually does. She put it right in front of her and with a ruler and her pencil she drew a line right on my grid.

It looked like this:

Multiplication Facts

	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11
2	2	4	6	8	10	12	14	16	18	20	22
3	3	6	9						27	30	33
4	4	8		16					36	40	44
5	5	10			25				45	50	55
6	6	12				36	42		54	60	66
7	7	14				42	49		63	70	77
8	8	16						64	72	80	88
9	9	18	27	36	45	54	63	72	81	90	99
10	10	20	30	40	50	60	70	80	90	100	110
11	11	22	33	44	55	66	77	88	99	110	121

“As you can see, the square numbers form a diagonal. Do you know why, Kayla?” Ms. Gibbs asked.

I looked at the grid and - and I could see why - I think. “When you look at just the squared numbers, every time you increase the number going across by one, you have to increase the number going down by one. So the squared numbers form a slanted line from the top left to the bottom right,” I answered. “And I guess that’s what a diagonal is.”

“Yes, that’s exactly right, Kayla! Now for next week, I’d like you to learn some more multiplication facts. What about learning the rest of your four times facts?”

“*All* the rest?” I asked.

“Yes,” Ms. Gibbs replied, “You already know your two times facts so just make each answer twice as big to get the four times facts. It shouldn’t be too hard.”

“Make my answer twice as big?”

“Yes. Four is twice as big as two. You know that two times three is six so four times three is twice that, or twelve.” Then Ms. Gibbs handed me a small piece of paper and said, “Write out your four times facts and if you know them all next week, you’ll be able to put...let me see…five more numbers into your grid.”

Five more numbers? OK, I can do it. I already knew what four squared is and I already knew what four times ten is and four times eleven, too…so I only had to learn five new facts.

I wrote out the five equations without Ms. Gibbs telling me what the answers were. It was easy. I just made the answers twice as big as I would get if I multiplied the same numbers by two. So if two times three is six, then four times three must be twelve. Here's what my paper looked like when I finished:

$$4 \times 3 = 12$$
$$4 \times 5 = 20$$
$$4 \times 6 = 24$$
$$4 \times 7 = 28$$
$$4 \times 8 = 32$$

"Kayla, you probably will have your multiplication grid filled out by the time you finish fourth grade," Ms. Gibbs said as she put my grid into my folder.

Oh no, I didn't get a chance to count the number of boxes I had left!

Reader, please count the number of empty boxes left in Kayla's grid, and write it here: ______ *The grid is back on page 14.*

Chapter 4

Dividing fractions

“Today I’m going to show you how to divide fractions,” Ms. Gibbs said.

“Divide fractions? But Ms. Gibbs, you already showed me how to divide fractions,” I said, “Remember?”

Ms. Gibbs looked puzzled. “Why no, Kayla, I don’t think I did.”

Hmm. I think Ms. Gibbs must have forgot but that’s OK, sometimes I forget things too. She always helps me remember, so now it’s my turn to help her remember.

“Don’t you remember?” I asked. “You said that dividing makes the numbers in the fraction smaller,” I explained. Here I’ll show you:

$$\frac{\mathbf{2 \div 2}}{\mathbf{4 \div 2}} = \frac{\mathbf{1}}{\mathbf{2}}$$

See what I mean?

“Kayla, I can understand why you might think you divided by a fraction. You *did* divide both the numerator and the denominator by the same whole number. That made the numbers in the fraction smaller without changing the value of the fraction. You *simplified* the fraction. That’s not the same as *dividing* by a fraction,” Ms. Gibbs explained.

“Huh?”

“Kayla, let’s think about what dividing does. First of all, let’s use the correct mathematical terms. We first used them when we studied improper fractions and mixed numbers. The number that’s being divided is called the ‘dividend,’ and the number that’s doing the dividing is called the ‘divisor.’ What dividing does is tell us how many divisors there are in the dividend. Sometimes there’s a left-over, and it’s called the ‘remainder.’

“If there’s nothing left over, we say that the divisor goes into the dividend ‘evenly.’ Ten (the dividend) divided by two (the divisor) tells us how many twos there are in ten: There are five twos in ten. Two goes into ten evenly, so there’s no remainder.

“Ten divided by three tells us there are three threes in ten, which make nine, so there is one left over. Three does not go into ten evenly. Ten divided by three is three, with a remainder of one.

“Let’s use cards for an example. Suppose we had ten cards and wanted to split them up into piles with two cards in each pile. If we wanted to know the number of piles of two cards there would be, we could use simple division:

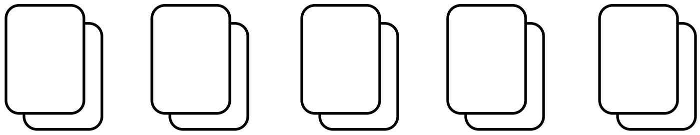
$$\mathbf{10 \div 2 = 5}$$

“Or we could write this division problem as an improper fraction:

$$\mathbf{\frac{10}{2} = 5}$$

“Either way we do it, our answer will be the same: There are five twos in ten, so there would be five piles of two cards each.

“Here’s a picture of the piles:



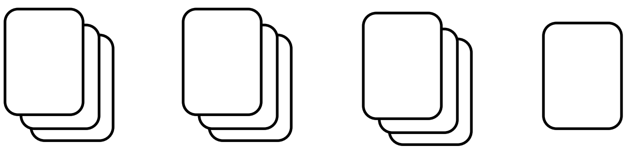
“Now suppose we want to split our ten cards into piles with three cards in each pile. Our improper fraction would be:

$$\frac{10}{3}$$

“Ten divided by three is three, with a remainder of one. You remember that to convert this improper fraction to a mixed number, you divide ten by three, and put the remainder over the divisor:

$$\frac{10}{3} = 3\frac{1}{3}$$

“That means we would have three and one-third piles. Each pile has three cards, and the one-third of a pile is just one card. And here’s what that might look like:



Reader, if you want to review converting improper fractions to mixed numbers, look at page 30 in Book 6.

"I understand what you did," I said, and then added, "When I was taking pizza orders at Cleveland's house last week, I had to convert two improper fractions into mixed numbers. I did it right - at least I *think* I did it right, although the pizza man didn't… Well anyway, that was real life, so now I'm sure fractions are for real."

"Yes, Kayla, fractions are for real, and I'm very glad you used the math you're learning here in 'real life.' You'll find that you'll use math in 'real life' more and more. Those who don't understand this basic math may run into trouble."

I didn't understand what Ms. Gibbs meant when she said, "run into trouble," but I didn't ask her. Maybe I will another time. Right now, I'm just going to focus on dividing by a fraction.

"Now let's get back to your lesson. Since you're learning about dividing *fractions*, we'll start out using the improper fraction way rather than the simple division way to solve this next problem:

"How many quarters are there in one dollar?"

I was going to say, "Four," but Ms. Gibbs didn't wait for my answer.

"Since a quarter is one-fourth of a dollar," she said, "asking 'How many quarters are there in one dollar?' is the same as asking 'How many one-fourths are there in one?' How would you set up the equation?"

Hmm. I know there are four quarters in one dollar, but...

Again Ms. Gibbs didn't wait for me to answer. She just continued, "When I wanted to find out how many twos there are in ten, I wrote:

$$\frac{10}{2}$$

Two is the divisor. Well, if I wanted to find out how many one-fourths there are in one, what would the divisor be?"

"One-fourth?" I answered tentatively.

"Yes, of course, one-fourth is the divisor and that means you need to divide by a fraction:

$$\frac{1}{\frac{1}{4}}$$

This asks the question: 'How many one-fourths are there in one?' And to answer the question, you'll need to divide one by the fraction, one-fourth."

Hmm, now I can see that this is a whole lot different from just simplifying fractions. This is new stuff for me. I better focus.

"Kayla, there is a very simple rule for dividing by a fraction and here it is: When dividing by a fraction, first invert it, which means 'turn it upside-down' and then *multiply* by it. That's all there is to it."

UPSIDE-DOWN!? I have to turn a fraction UPSIDE-DOWN? That's what mixed up my momma when she was in school! I better pay attention to this so I'll be able to explain it to her.

"When we turn one-fourth upside-down, it becomes four over one, which of course is just four, so one *divided* by one-fourth is the same thing as one *multiplied* by four." Then Ms. Gibbs wrote out what she meant:

$$\frac{1}{\frac{1}{4}} = 1 \text{ X } \frac{4}{1} = 1 \text{ X } 4 = 4$$

So there are four 'one-fourths' in 'one' and four quarters in one dollar, which you knew already. But this simple example is a good way to introduce the idea of dividing by a fraction.

"In fact, this rule, 'invert and multiply,' works even for dividing by a whole number! You remember that any whole number can be written as a fraction with a denominator of one, for example:

$$2 = \frac{2}{1}$$

So to divide four by two, we can write:

$$4 \div 2 = 4 \div \frac{2}{1} = 4 \text{ X } \frac{1}{2} = \frac{4}{2} = 2$$

Of course you knew that four divided by two is two. I just wanted you to see that the rule 'invert and multiply' works for whole numbers as well as fractions.

"We can divide fractions by whole numbers using this rule, too. Suppose we have three-fourths of a pizza, and we want to split it equally among six kids. We need to divide three-fourths by six.

I'll set up the problem for you, but first let me write the whole number, six, as a fraction, six over one:

$$\frac{3}{4} \div \frac{6}{1}$$

Now, Kayla, do you think you can solve this division problem using the 'invert and multiply' rule?"

I nodded my head. I think I can do it. First I invert the six over one, and then I just multiply across, and then I'm done:

$$\frac{3}{4} \times \frac{1}{6} = \frac{3}{24}$$

I looked at my answer, then said, "Oh, wait! I'm *not* done. I have to simplify the fraction three-twenty-fourths to one-eighth:"

$$\frac{3 \div 3}{24 \div 3} = \frac{1}{8}$$

"I can now see that simplifying is a whole lot different than dividing by a fraction."

"That's right, Kayla, so how much pizza does each student get?" Ms. Gibbs asked.

"Huh?"

"Kayla, you divided the amount of pizza you started with, three-fourths, by the number of students who would get a piece, six, and you learned that each student would get one-eighth of a pizza."

"Oh," I said, "I remembered the 'invert and multiply' rule, but I forgot what I was trying to figure out!"

Then Ms. Gibbs asked me to prove that one-eighth is the correct answer. How am I going to do that? I picked up my pencil, but I didn't know what to write.

“Kayla, division and multiplication are inverses of each other. So if you multiply the amount of pizza each student gets by the number of students, you should get the total amount of pizza you started with,” Ms. Gibbs explained.

I picked up my pencil again. This time I wrote the amount of pizza each kid gets, one-eighth, and multiplied that by the number of kids, six, and and got six-eighths:

$$\frac{1}{8} \text{ x } 6 = \frac{6}{8}$$

“Oh, I can simplify this:

$$\frac{6}{8} = \frac{3}{4}$$

Hey! That’s the amount of pizza I started with. I was surprised but I tried not to show it when I said to Ms. Gibbs, “I just proved that I did everything right!”

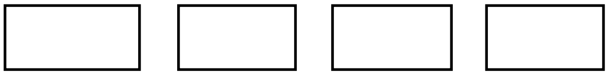
“Yes you did, Kayla, that’s very good! You can see that this rule is very convenient, and it’s not hard to use.”

“Yes, because I already know how to multiply fractions. And dividing by a fraction is the same thing except I turn the second fraction – that’s the divisor - upside-down before I multiply.”

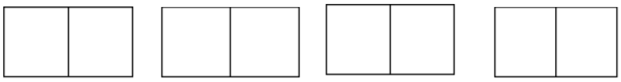
“It helps to work a lot of problems and to be able to visualize your answers,” Ms. Gibbs said. “Using fraction bars is a good way to do that.

“Let’s start with this problem: A Park Ranger gives tours and each one takes one-half hour. If we want to know how many tours she can give in four hours, we can solve this problem using fraction bars.

“Since there are four hours, we’ll start with four fraction bars, one for each hour:



Each tour takes one-half hour, so if we split each hour into two parts, we have:



For a total of eight tours.

“When we solve this problem using math, Kayla, the first thing you have to do is figure out what this question, which is *in words,* means when we express it *in math*. Can you do that?”

I thought about it a bit, and then said, “’How many tours?’ is the same as ‘How many one-half hours?’ because that’s how long each tour takes. Four is the total number of hours, so we want to know how many ‘one-halves’ there are in four.”

“Exactly right, Kayla,” Ms. Gibbs said, “Now can you write that down using numbers?”

I didn’t say anything. I just wrote down:

$$\mathbf{4 \div \frac{1}{2}}$$

“Good for you, Kayla!” Ms. Gibbs said, “and what do you do next?”

“I just invert the one-half and multiply, right?” I asked. But I didn’t wait for an answer. I turned the two upside-down and wrote a plain old two, and four times two is eight:

$$4 \times 2 = 8$$

“That’s the same answer we got with the fraction bar. So the Park Ranger can give eight tours in four hours. This really is not hard at all,” I said, “once I change the question given *in words* into a *math* question.”

“You’re learning fast, Kayla! Here’s another problem,” Ms. Gibbs continued, “Suppose we are making cookies, and one batch of cookies requires two-thirds of a cup of sugar. We have plenty of all the ingredients except sugar. We have only two cups of sugar. How many batches of cookies can we make?

“We can represent the two cups of sugar we have as fraction bars, and since we are talking about ‘thirds’ of a cup, we’ll divide each fraction bar into three equal sections:

“Each section is one-third of a cup, so two-thirds of a cup is two sections. It is easy to see that if we fill in two sections of each fraction bar, there will be one section left over in each. But that’s another two ‘thirds,’ making the total number of two-thirds equal to three.

Reader, please fill in two sections in each fraction bar so you can see there is a total of three “two-thirds of a cup” in two cups.

“Kayla, can you now show me how to solve this example using math instead of fraction bars?”

“I think so,” I said. “There are two cups of sugar, and for each batch of cookies we need two-thirds of a cup, so we have to find out how many ‘two-thirds’ there are in two,” I said, “that’s just two divided by two-thirds,” and I wrote down:

$$\mathbf{2 \div \frac{2}{3}}$$

“When we invert the fraction and multiply, we get:

$$2 \ \mathbf{x} \ \frac{\mathbf{3}}{\mathbf{2}} = \frac{\mathbf{6}}{\mathbf{2}} = \mathbf{3}$$

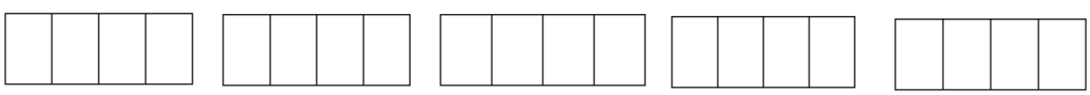
“So, just the same as with the fraction bars, we can make three batches of cookies with two cups of sugar. Wow! You better know your math if you’re a cook!”

“That’s certainly true, Kayla. And it’s true for lots of other jobs, too.

“Here’s another problem that can use fraction bars to help understand it, but it’s a little harder:

“Suppose you want to cut bookmarks from a piece of cardboard that is five inches wide. Each bookmark should be three-fourths of an inch wide. How many bookmarks can you make?”

”You already know that you can get the answer by finding out how many three-fourths there are in five using our rule, but I want you to use fraction bars to help you understand the problem. We can let each inch of cardboard be a fraction bar, and divide each into fourths. This is what it looks like:



"Now, you know that each bookmark uses up three-fourths of an inch. The first one uses up the first three boxes of the first fraction bar, so I'll write a "1" in those three boxes," Ms. Gibbs said as she wrote three ones in the first three boxes of the first fraction bar:

1	1	1																	

"Kayla, do you think you can figure out how many bookmarks you can make using the rest of these fraction bars?"

Reader, do you think you can figure it out? Try to do it before you read any more.

Hmm… Now let's see. Ms. Gibbs put a "1" in the first three boxes because the first bookmark took up three-fourths of an inch. The next bookmark, the *second* one, will take up three-fourths of an inch also, so I could put a "2" in the next three boxes. I started writing numbers in the boxes until there were no more boxes left. Hey! I figured it out! That would be the same as if I ran out of cardboard for making bookmarks!

This is what the fraction bars looked like when I finished:

1	1	1	2	2	2	3	3	3	4	4	4	5	5	5	6	6	6	7	7

I could see that there was enough cardboard to make six bookmarks, but not enough to make seven. We need three-fourths of an inch per bookmark, and there are only two-fourths available after the sixth one.

So I said to Ms. Gibbs, “It looks like we can make six bookmarks. There is some cardboard left over, but not enough for another bookmark.”

“That’s very good, Kayla! You have understood the problem and used fraction bars to solve it. Now let’s see you solve the same problem using math.”

“The first thing I have to do is know that this question is just asking how many ‘three-fourths’ there are in five,” I said, “and that’s just five divided by three-fourths:

$$5 \div \frac{3}{4}$$

“Now I just invert the fraction and multiply:

$$5 \times \frac{4}{3} = \frac{20}{3} = 6\frac{2}{3}$$

“That’s the same answer I got with the fraction bars: I can make six bookmarks with five inches of cardboard, and there is only enough cardboard left over to make two-thirds of a seventh one.

“The fraction bars make it easy to understand, but using the ‘invert and multiply’ rule is much faster.”

“That’s true, Kayla,” Ms. Gibbs said.

I can hardly wait to tell my momma about upside-down fractions! And tell her why, too. She’ll want to know why. It’s because... hmm, I didn’t know why. I looked up at Ms. Gibbs but before I got to ask her “why,” she continued:

“I’d like to show you why this rule works, Kayla. This will be the first time we have looked at a mathematical proof. It may seem a little difficult but don’t worry. You won’t actually have to *use* the proof, but it is good to know where the rule came from.”

Reader, don’t worry. You won’t have to use this proof – or even understand it – but if you can, good for you! Ms. Gibbs just wanted to explain it to me.

“Mathematical proof?” I exclaimed. Gee, that sounded hard, but Ms. Gibbs said I wouldn’t need to actually use it. I still tried to understand what she was saying.

Ms. Gibbs began by saying, “Let’s suppose we want to divide two by three-fourths:

$$\frac{2}{\frac{3}{4}}$$

“We want to show that this is the same thing as two multiplied by four-thirds:

$$2 \text{ x } \frac{4}{3}$$

“Now we’re going to divide four-thirds by four-thirds. As you know, Kayla, anything divided by itself equals one, so:

$$\frac{\frac{4}{3}}{\frac{4}{3}} = 1$$

“And you know that multiplying anything by one doesn’t change its value, so two divided by three-fourths times four-thirds over four-thirds is still two over three fourths:

$$\frac{2}{\frac{3}{4}} \quad \text{x} \quad \frac{\frac{4}{3}}{\frac{4}{3}} \quad = \quad \frac{2}{\frac{3}{4}}$$

“Now we focus on the left-hand side of the equal sign. It looks complicated because there are fractions within these two fractions. But they are still just fractions, and follow the rules for fractions. So we just multiply across, both numerators and denominators, and now the left-hand side looks like this:

$$\frac{2 \text{ x } \frac{4}{3}}{\frac{3}{4} \text{ x } \frac{4}{3}}$$

“Here the new numerator is two times four-thirds:

$$2 \text{ x } \frac{4}{3}$$

“And the new denominator is three-fourths times four-thirds:

$$\frac{3}{4} \text{ x } \frac{4}{3} = \frac{12}{12} = 1$$

“Because the denominator is one, we don’t have to write it down, so the result of multiplication on the left-hand side is just:

$$2 \times \frac{4}{3}$$

“But this must be equal to the right-hand side, which is:

$$\frac{2}{\frac{3}{4}}$$

“This result means that two *divided* by three-fourths is the same thing as two *multiplied* by four-thirds! That’s exactly what we started out trying to prove. Since we could have chosen any number for the numerator, and any fraction for the denominator, this must be true for *any* numbers and fractions. That’s what makes it a rule.”

“Oh.” That’s all I said. I tried to follow along with what Ms. Gibbs was doing, but I kept getting lost.

I think Ms. Gibbs knew that I didn’t understand much of what she did, because she said, “Not many people can easily understand this proof the first time they see it.”

Then Ms. Gibbs told me, “A very long time ago, mathematicians figured this out and made a simple rule for anyone who wants to answer questions such as ‘How many one-fourths are there in two?’ which is the same thing as ‘What is two divided by one-fourth?’

“And the rule is this: ‘To divide by a fraction, just invert it and multiply.’ Rules like this are sometimes called ‘algorithms.’
“For you, Kayla, simply being able to apply the rule, ‘invert and multiply,’ is all you need to know for now. If you’d like to be a math teacher someday, you will learn more about proofs.”

“OK,” I said, “But for right now, I think I’ll just use the ‘invert and multiply’ rule.

Ms. Gibbs said, “That’s what I do, too.”

Reviewing what I learned

It's time to review how to divide fractions. The rule is very simple: invert the divisor and multiply. And invert just means to turn the divisor upside-down. That's pretty easy, isn't it?

But what's not so easy is to know which fraction is the divisor! If you don't know that, you won't know which one to turn upside-down and if you turn the wrong one upside-down, well then, your answer will be all wrong.

One way to help you figure this out is to change the word question into a math question. Ms. Gibbs told me about this the very first day I met with her and it's something I always use and I hope you do too.

Here is what she said: "A math question is just a question that uses numbers and symbols instead of words." Symbols are things like this:

$$+ \; , \; - \; , \; \mathrm{X} \; , \; \div \; , \; =$$

It's funny because I don't even think of these symbols as symbols any more. I think of them as words. So when I see "=", I just say "equals," and when I see "X", I just say "times," as if those words are there, but they're not. I bet you do the same thing.

Well, anyway, Ms. Gibbs changed this word question for me: "How many quarters are there in one dollar?" into a math question: "What is $\mathbf{1} \div \frac{\mathbf{1}}{\mathbf{4}}$?" and then I knew how to solve it. I first inverted the one-fourth - that's the divisor - and then I just multiplied.

Not all the words in a word problem may be important to do the math. Take this problem for example: Suppose my teacher wanted to make posters to hang up in the cafeteria. Each poster is supposed to be three-fourths of a foot wide. He has five feet of posterboard. How many posters can he make?

When doing the math, it doesn't matter who's making what - you or your teacher, or bookmarks or posters, and it doesn't really matter if it's inches or feet. What does matter is the math question, and it's this: How many $\frac{3}{4}$ are there in 5? And that's five divided by three-fourths.

Does that sound familiar? If it does, that's because it's *exactly* the same math question that Ms. Gibbs asked me on page 27, but in that problem, I was making bookmarks and using inches. But the math is the same, really it is! If the math is the same, the answer must be too. It's $6\frac{2}{3}$.

Words, however, do matter because you have to answer the question in words. When the problem is about bookmarks, I would say, "I can make six bookmarks but there's only enough cardboard left over to make two-thirds of a bookmark." Now if I answer the question about the posters, I would say, "My teacher can make six posters but there's only enough posterboard left over to make two-thirds of a poster." Got it? You use the same numbers, so the math is the same, but what you figured out is different.

There is one more thing I'd like to tell you about that bookmarks example Ms. Gibbs and I did. I was confused about it at first, but Ms. Gibbs explained it to me afterwards:

First we solved for the number of bookmarks using five fraction bars with four boxes in each, remember?

Each box was one-fourth of an inch, and we knew that each bookmark used up three-fourths of an inch. I put ones in the first three boxes, that was one bookmark; then twos in the next three boxes, that was the second bookmark; and so on. I ran out of boxes after putting two sevens in the last two boxes.

I told Ms. Gibbs we could make only six bookmarks, because we need three-fourths of an inch for each one, and there were only two-fourths available for the seventh one.

But when we did the math using the ‘invert and multiply’ rule, our answer was six and two-*thirds* bookmarks. Why did the example with boxes talk about “fourths” and the math calculation talk about “thirds”?

The answer is that in the example with the boxes, we were talking about *inches* and the math calculation told us the number of *bookmarks*. Since each bookmark takes up *three*-fourths of an inch, two-thirds of a bookmark is two-thirds of three-fourths of an inch, which is just *two*-fourths of an inch. That means *two*-thirds of a bookmark must be the same as *two*-fourths of an inch! Got it?

This shows how important it is to know whether we’re talking about inches or bookmarks. Two-thirds is definitely not the same as two-fourths when we are just talking about numbers:

$$\frac{2}{3} \neq \frac{2}{4}$$

But it is true that:

$$\frac{2}{3}\text{ bookmark} = \frac{2}{4}\text{ inch}$$

Two-thirds of a *bookmark* and two-fourths of an *inch* are not pure *numbers*, like just plain two-thirds and plain two-fourths are. When we tack on units, like bookmarks and inches, that changes things. Just like the number, $\mathbf{1}$, doesn’t equal the number, $\mathbf{5}$, but

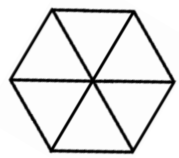
1 nickel *does* equal **5** pennies. Ms. Gibbs told me we would talk more about this at another time.

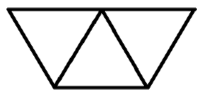
Before I finish this review, I want to show you one more good way to visualize dividing by a fraction, this time using hexagons.

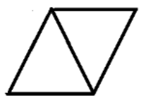
Remember when Ms. Gibbs used hexagons to help me understand *adding and subtracting fractions*? It was in Book 6, starting on page 28.

You can use hexagons to help you understand *dividing by a fraction,* too. I'll do a little review for you. It's always good to review.

Remember that we can divide a hexagon into six, three or two equal parts.

You can think of a whole hexagon as six triangles: 

one-half of a hexagon as three triangles: 

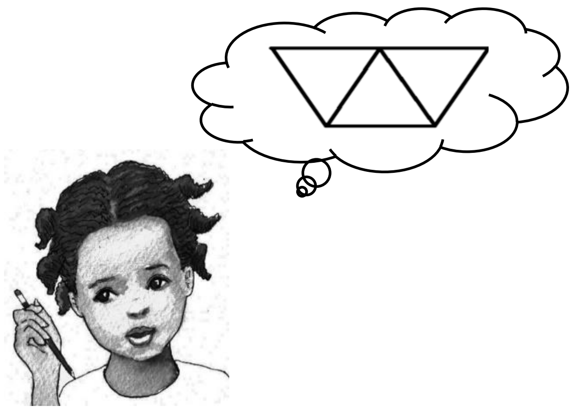
one-third of a hexagon as two triangles: 

and one-sixth as just one triangle: 

Now see If you can solve this problem without using our "invert and multiply" rule:

$$\frac{1}{2} \div \frac{1}{6}$$

If you just think of these as numbers, it might seem like a hard problem. But if you think of these fractions not as *numbers*, but as *hexagon parts*, you can solve it very easily. The question you want to answer is: ‘How many one-sixths are there in one-half?’”



All you have to do is look at the pictures of one-half of a hexagon and one-sixth of a hexagon. Since one-half a hexagon is made up of three triangles, and each triangle is one-sixth, there are three one-sixths in one half. You don’t have to calculate *anything*!

$$\frac{1}{2} \div \frac{1}{6} = \mathbf{3}$$

Ms. Gibbs says hexagons help us visualize fractions. “Visualize” means to see, and you can *see* there are three triangles in one-half of the hexagon.

If you use the “invert and multiply” rule to solve the problem, this is what it would look like:

$$\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \text{ X } \frac{6}{1} = \frac{6}{2} = \mathbf{3}$$

You get the same correct answer, but you may not really understand things the way you can when you use hexagon parts. And understanding is always very helpful.

After you've done enough problems, you'll have dividing by fractions down-pat.

That's what I like about math so much. Once you really understand how to do the math, you can apply it to so many different problems. But you *gotta* know how to do it - and to really understand it.

Don't let the words in the problem get you mixed up. I think this is *so* important, I'm going to write a whole book about it and I'm going to get an expert problem-solver to help me write it. He's already written lots of books about problem solving but he said he would write a book with me. Imagine that! It will be Book Twelve. Make sure you get it!

Another thing that's important for you to know is this: Everyone learns math (and other stuff, too) a little differently. In the problem about the bookmarks, I found putting numbers right in the fraction bars helped me really understand what I was doing. Otherwise, I might have gotten the correct answer and my teacher or tutor might say I'm right, but I wouldn't really be confident that I was right. It's nice to feel confident.

And please don't worry if other kids understand things faster than you do. It's not important how fast you learn stuff, what's important is that you learn it. If you get stuck with something, try to figure out what you don't know, and then ask someone who knows to help you learn it.   And make sure you practice your math. You *gotta* practice if you want to be good in anything - especially math.

There is one more thing I have to tell you before I finish this chapter and it’s this: there is lots of pretending you can do with math, and I like to pretend. I pretended that I was making bookmarks for my school books, and I pretended that Mr. Williams was making posters for the cafeteria. Hmm... I wonder what was on the posters. Maybe it was about a party – maybe it was a pizza party… what do you think?

Practice problems

Don’t worry, I’m not going to ask you anything about that proof. I really didn’t understand it very well at the time myself. But you do have to remember the rule for dividing by a fraction. It’s easy: just turn the divisor upside-down and multiply!

The first few problems are similar to the examples in Chapter 4, but there *is* a difference. Try to do them without looking back at how I did those similar problems. But if you have trouble with any of them, that’s OK, just go back and review what I did. And if you want more practice, you can find similar problems on my website!

1. Suppose you have ten cards and you want to divide them equally among five friends. How many cards does each friend get? Hint: Ask yourself what the divisor is.

_____ cards

2. How many quarters are there in five dollars?

_____ quarters

3. Suppose you want to make bookmarks by cutting them from a piece of cardboard. Each one must be three-eighths of an inch wide, and the total width of the piece of cardboard is ten inches. How many bookmarks can you make? If the answer is not a whole number, write it as a mixed number like I did on page 29.

________ bookmarks

4. Suppose you have one-half of a pizza left over from a party. You want to divide it evenly among three friends and yourself. What fraction of a pizza does each of you get? Prove that your answer must be correct. (see page 24)

______ of a pizza

5. You have three bananas, and you want to share them equally among 5 people. How much does each one get?

______ of a banana

6. Five of your friends are coming over to your house for breakfast. You want to make pancakes for them and yourself. The recipe says you need three-fourths teaspoon of vanilla to make one batch of eight pancakes. You have plenty of all the ingredients except vanilla. You have only two and one-fourth teaspoons of vanilla. How many batches of pancakes can you make? How many pancakes does each person get? (see page 26)

_____ batches

_____ pancakes each

7. Watch the signs and make sure you simplify whenever possible. If the answer is an improper fraction, convert it to a mixed number:

a) $\frac{1}{2} \div 4$ b) $4 \times \frac{1}{4}$

c) $\frac{1}{2} + \frac{1}{3}$ d) $\frac{7}{9} - \frac{1}{3}$

e) $\frac{1}{2} - \frac{3}{7}$ f) $\frac{3}{4} \times 6$

g) $\frac{2}{3} \div \frac{2}{5}$ h) $9 \div \frac{1}{8}$

8. Just multiplication:

a) $50 \text{ x } 2000 =$ __________ b) $1000 \text{ x } 750 =$ __________

c) $90 \text{ x } 900 =$ __________ d) $102 \text{ x } 600 =$ __________

e) $8654 \text{ x } 92 =$ __________

9. Your whole class, 24 students, is going on a field trip. Your teacher plus two parents will be going also, as chaperones. The vans available hold ten people each, not counting the driver. If you are the one who has to reserve the vans, how many should you reserve? If there are any empty seats in one of the vans, how many would there be?

_____ vans

_____ empty seats

10. Remember this problem? Alfa got the right answer and won a bike! I changed the numbers to make it a little different.

A big store had a contest for its workers. The owner brought all the workers into the storeroom and showed them six shelves full of jars of jelly beans. There were eighteen jars on each shelf, and each jar contained two hundred jelly beans. What is the total number of jelly beans in the storeroom?

______ jelly beans

Something extra

“Algorithm” is the new word I have for you. Isn’t that a neat word? There are four syllables. The first two are pronounced “al-guh” with the accent on “al.” The last two are pronounced just like the word “rhythm,” in music.

An algorithm is just a set of instructions. Usually there are a lot of instructions in an algorithm, but for dividing by a fraction, there are only two:

1. invert the divisor
2. multiply

“Algorithm” is a very old word. It comes from the name of a 9th century mathematician named Muhammed ibn-Musa al Khwarizm. It’s not important to remember his name - I have trouble even pronouncing it.

He lived in Baghdad, in a country now known as Iraq. Some of his mathematical procedures were named after him, and as the centuries passed by, his name, “al Khwarizm,” gradually got changed into “algorithm,” to mean “a set of instructions.”

It is important is to realize that people from different countries all around the world have made important contributions to the development of mathematics.

Answers

1. 2

2. 20

3. $26\frac{2}{3}$

4. $\frac{1}{8}$

5. $\frac{3}{5}$

6. 3 batches, 4 pancakes each

7a. $\frac{1}{8}$ 7b. 1 7c. $\frac{5}{6}$ 7d. $\frac{4}{9}$ 7e. $\frac{1}{14}$ 7f. $4\frac{1}{2}$

7g. $1\frac{2}{3}$ 7h. 72

8a. 100,000 8b. 750,000 8c. 81,000 8d. 61,200

8e.
$$\begin{array}{r} 8654 \\ \underline{\times 92} \\ 17308 \\ \underline{77886} \\ 796168 \end{array}$$

9. 3 vans, 3 empty seats

10. 21,600

About Tutoring Math

If the persons you're tutoring have trouble understanding the math you're helping them with, don't get discouraged. Try to explain the math in a slightly different way. Give lots of examples and work lots of problems. And review frequently!

Think of little games to play. Dividing cards into piles, as was done on page 19, is a good way to teach what dividing means, whether it's fractions or whole numbers.

Everyone learns a little differently, and some may take longer - a lot longer - than others. The important thing is <u>not</u> how long it takes to learn something; the important thing is to learn it - and to really understand it.

Remember that the math a person learns early on will be used for the math taught later on. If the student you're tutoring learns basic math, s/he will have a much easier time learning the harder math taught in middle school, high school and in college. The work you're doing is VERY important to the student's success in school.