Learning Math with Kayla

Book 11 Learning about negative numbers

Vicki Meyer

The Learning Math with Kayla Books

Book 1 Adding and subtracting like fractions
Book 2 Multiplying fractions
Book 3 Learning multiplication facts
Book 4 Place values, Multiplying large numbers
Book 5 Adding and subtracting unlike fractions
Book 6 Learning about improper fractions and mixed numbers
Book 7 Dividing fractions
Book 8 Adding and subtracting large numbers
Book 9 Solving long division problems
Book 10 Working with decimals and per cents
Book 11 Learning about negative numbers
Book 12 Problem-solving!

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About the Kayla Books

The Kayla books tell the story of a fourth-grade girl who has gotten so far behind in her math class that she is not able to understand what her teacher is trying to teach her. Her math teacher, Mr. Williams, is aware of how poorly Kayla is doing. He decides a tutor would be the best way to help Kayla learn her math.

In this eleventh book, Ms. Gibbs, her math tutor, teaches Kayla about negative numbers. She uses examples of credit card debt and subzero temperatures to explain how everyday numbers can be negative. Ms. Gibbs also uses the number line to help Kayla add and subtract numbers, some of which may be negative. There are twelve books in this series.

Whether you're a fourth-grader, in middle school or in high school, a Mom or Dad or a Grandparent, you can learn along with Kayla.

The story is told by Kayla, right before she goes off to college.

About Kayla

I have been asked if Kayla is a real person. She and others in the books are composites of the many kids I have tutored, plus friends. And I include myself for this book, because I remember when I studied negative numbers in school I found some things confusing. At first I didn't know there was such a thing as "negative numbers." Once you counted backward to zero, I thought that was it. There wasn't anything less than zero. Well there is, of course, and that's what Kayla learns about in this book.

The Author

After Vicki raised six really smart kids, she began studying for her Ph.D. in order to keep up with them. She taught at the university level for 25 years, then began tutoring elementary school students. Vicki soon found a new career for herself, tutoring math for at-risk kids, writing about her experiences, and putting together the Kayla books. You can contact Vicki by going to the website (**www.learningwithkayla.org**) and clicking on "Contact."
Vicki lives with her husband, Ed, in Sarasota, FL.

Acknowledgements

Thanks to Charles Daniel for the excellent job he does in proofreading the text. His expertise in mathematics and grammar ensures that both kinds of errors are minimized in the Kayla Books.

And a special thanks to my husband, Ed, for all of his great suggestions, his skillful editing, and especially his patience. I would not be able to complete the books without him.

DEDICATION

To my mother, Phyllis Hurtova, who was prevented from going past the fourth grade due to political unrest in Czechoslovakia, but continued to be a life-long learner.

Table of Contents

Chapter 1

Very early in the morning

It was still dark outside. I tried to go back to sleep but I couldn't - I just couldn't and I know why. I was too excited! You see, today is the day of our big game. We're going to have a real basketball game today. It will be at 10 o'clock in the morning. It will be a real game with sides and scores and everything. That's what Cleveland said!

Oh-oh, I hear my Momma getting up. I know she has to go to work today. On the days she works, she leaves for the hospital even before I get up - that's where she works - the hospital. If it's a school day, I have breakfast at school. If it's a day when there is no school. I just have cereal by myself. But if it's a day that I don't have school and Momma doesn't have to work, then momma makes omelets for us. Those days are the best!

But today is Saturday, there is no school so I'll just have cereal by myself. I don't mind. I looked over at my clock, now it was just a couple of minutes after six. Oh no, the game is not until ten! I should go back to sleep but I can't. I'm trying but I just can't.

I heard Momma moving around her room. She must be getting dressed. Just a little bit later, Momma opened the door to my room. I could hear her walking to my bed. I kept my eyed closed. She very gently kissed me on my forehead. I still kept my eyes closed. You see, I was pretending I was asleep. Shh.

And then Momma quietly opened my drawer and just as quietly closed it. I was going to take a peek but I didn't. I wondered what she was doing. And then the door to my room open and closed and … and then I heard the front door open and close. Momma left for work!

I couldn't go back to sleep. My eyes just kept opening up. So I got out of bed. Right away I saw that my basketball shirt, you know, the one with the numbers on the back, was on the top of my dresser.

Oh, Momma must have taken the shirt out of my drawer. So that's why she opened my drawer   She knew all about my big game and she knew I would want to wear that shirt.

I remember when I was at the store, I wanted a shirt with numbers on the front and back but all they had in my size were ones with the number in the back. So that's what my momma bought for me.

Right away I put my basketball shirt on and looked in the mirror. Hey, I look like a real basketball player – well maybe just a little, but I did!

I quickly ate my cereal. Then I washed my bowl and spoon and put the cereal away. And then…I waited and I waited and I waited…until it was time for me to leave the house. While I was waiting, I tried to study a little math but I had trouble concentrating. You see, my priority this morning was the big game!

I wondered who would be at the game. I knew Cleveland would be there because - well because he's the one who organized it. It was just a little before 10 AM; finally it was time for me to go. There was no time left for me for wondering. But while I was walking to the playground, I still wondered. I wondered who would be on my team.

As I was walking, I saw Liz and Luz walking ahead of me. I called to them and they waited for me. We walked to the playground together.

When we got to the playground, Cleveland and his brother, Jerod, were already there. They were practicing. Jerod greeted us with a big “Hi” and a wave of his hand. He was as excited as I was.

Cleveland just nodded his head. Then I heard Cleveland yell, “And there’s Jake!” Then he yelled to Jake, real loud, “Jake, over here,” and Jake began walking toward us.

When I first saw Jake, I thought to myself, Oh, no, that’s - that’s that mean boy from school and he’s real big. I think he might have had to repeat a grade because he’s much bigger than I am. Why he’s even bigger than Cleveland!

As soon as Jake came over, he yelled out real loud, “Oh no! Girls! I’m not playing with no girls!” And he said it real mean-like!

At first I was going to correct his English but then I thought, maybe I better not!

Reader: Do you know what English Kayla was going to correct? The correction that Kayla made is in Answers.

I didn’t know what to do so I just backed up a bit. Should I leave? Well, that’s what I did - well that’s what I sort of did. I walked slowly away, but backwards – you see, I wanted to see what was going to happen.

Well, I’ll tell you what happened! Liz didn’t move, not even a tiny bit. She just had her hands on her hips, the very same way she did when Cleveland knocked her chair over. Luz was standing next to Liz - she had a big frown on her face. Her arms were folded in front of her.

Then Cleveland yelled and he yelled it real loud, “They’re staying!”

Right away, I stopped walking backwards. I just stood there. I was watching to see what was going to happen next. Then - all of sudden - Jake turned around, stood there for just a bit, then started walking slowly back toward us.

Cleveland yelled out, “OK we’re playing!” And then he said, “I’ll take my little brother and… and…I’ll take Kayla. Liz and Luz and Jake, you’re on the other team.” Liz and Luz didn’t say anything, they just walked over to Jake.

I’m so glad I’m on Cleveland’s side and I’m so, SO glad I’m not on Jake’s side. Liz and Luz didn’t seem to mind being on Jake’s team though. And then Cleveland yelled, “OK, let the game begin!”

Chapter 2

The game

Cleveland had the ball first and he passed it to me. I passed the ball to Luz. Oh no! I shouldn't have done that! Luz is on the other side! Now she has the ball and…and…and she made a basket! Oh no! They got the first two points and it was all my fault!

I was afraid Cleveland would yell at me but all he said was, "It's OK, we'll get the next points." And guess what: we did! Cleveland made a basket!

Next time I received the ball I passed it to Jerod. He's on my side, *that* I remembered. Jerod passed the ball to Cleveland and Cleveland passed it to me. Why is Cleveland passing the ball to me? I quickly passed it back to Cleveland and Cleveland tried to make another basket but this time, the ball didn't go in the basket.

The ball kept going back and forth. I was trying not to get mixed up. I was so glad I practiced my dribbling because, you see, I was dribbling a lot - and I was pretty good. Not as good as Cleveland or Jerod - or Liz or Luz. They were really good, but I was pretty good too - at least for me I was. Jake was good too but that's because… well, that's because he's so big.

The game continued and I was remembering who was on my team. Luz had the ball and passed it to Liz. Liz attempted to make a basket but she missed. Cleveland got the ball and he dribbled to the other end of the court to make a basket but he missed too. The ball came to me! I didn't want it so I quickly passed it to Jerod. Jerod passed the ball to Cleveland and this time, Cleveland made a basket! The ball went right in the basket, it didn't even hit the rim. Cleveland called the score. It was 12 to 12!

We had to run back to the other side of the court really fast, because Jake didn't like that Cleveland tied up the game with that last basket. Jake wanted to make a basket right away so his team would be winning again. But he tried to go too fast and he tripped and fell down! The ball rolled away from him. But lucky for him Liz was running fast, too, and she was able to pick up the ball. She dribbled a couple of times and then shot the ball right into the basket! I think Jake started thinking right then that maybe playing basketball with girls wasn't so bad after all.

I couldn't help thinking that I'm a pretty good dribbler. I'm so glad I practiced so much. You have to practice if you want to be good at anything and I practiced dribbling a lot and now I'm a pretty good dribbler – well for me, I am.

I was getting a little dizzy watching the game and I couldn't remember the score but I *did* remember who was on my side. Every time the ball came to me I passed it either to Cleveland or to Jerod because – well, we're all on the same side. Then all of a sudden, the ball came right to me again. I was right under the basket. I heard Cleveland yell, "Shoot!"

So I did and you'll never guess what! The ball hit the rim first, I watched it - well everyone was watching it - and then the ball went right into the basket. Wow! I made a basket!!!

I made my very first basket! Everyone was watching and they all cheered for me - well it sure sounded like everyone was cheering. I remembered hearing Cleveland and Jerod cheering for me. Luz and Liz cheered for me too even though they were on the other side. Jake didn't really cheer for me but he was smiling, I saw him.

I couldn't believe I made a basket. The ball that I threw went right into the basket! And everyone saw it! I made my first basket and it was in a real game!

After everyone - well almost everyone - was done cheering for me the other team got the ball. I tried but I couldn't concentrate on the game anymore. You see, I kept thinking of my ball going into the basket. I can hardly wait to tell my momma and, let's see, who else will I tell? Oh, I'll tell Ms. Gibbs! Maybe I'll tell Mr. Williams too.

I better pay attention to the game. I don't even know what the score is. The other team kept passing the ball to each other, and then Luz attempted to make a basket and she did - the ball went right in and it didn't even hit the rim. I clapped for her, but just a little, even though I knew I shouldn't have clapped at all. You see, she's on the other side.

Oh, Jake made a few baskets too, but – but that's because he was so big. Pretty soon, Cleveland yelled, "Game over!" The score was 22 to 20. Jake's team won!

I didn't know what we were supposed to do next. It seemed like our team won but we didn't win, we lost.

What do we do now? I'll just follow what everyone else is doing.

We walked around shaking hands with the other team. When Jake shook my hand, he said in a low voice, but I heard him - "Hey, nice basket."

The other team looked real happy because they won. I was real happy too, but not because we won, we didn't, but because I made my very first basket - and it was in a real game. And that means the two points I made really counted.
Then I heard Cleveland yelled out "Rematch" and Jake said, "Any time." After Jake said that, Cleveland yelled, "And let's keep the same sides!"

Jake wants to play with us again and Cleveland does too and he wants to keep the same sides. That means I'll be on Cleveland's team again - although it might not be sooo bad if I was on Jake's team.

And then I started to think about my basket again. I was so happy I made my very first basket and it was in a real game. I got two points for my team. And we only lost the game by two points. If I made another basket, the score would be tied and if I made another basket after that, why, we'd win the game!

Chapter 3

Negative numbers

Finally, It's tutoring day! The big game was last Saturday and I had to wait until today, Thursday, to tell Ms. Gibbs about the game. I want especially to tell her about the basket I made. I'll tell her I made a basket, my first basket ever - and it was during a real game. I made two points for my team!

But I didn't get a chance because Ms. Gibbs said we had a lot of work to do. After we greeted each other, Ms. Gibbs said she wanted me to learn about negative numbers.

"Negative numbers?" I asked.

"Negative numbers are just like regular numbers but they have a negative sign in front, toward the top, like this: **⁻4.** You read this, 'Negative four'," she said.

"Negative numbers are less than zero," Ms. Gibbs explained. We use negative numbers in our everyday lives, so it's a good idea to understand them."

"We do?" I asked.

"Yes, Kayla, there are two common situations that use negative numbers. One involves money and the other involves temperature."

"Huh?" I said. "Money and temperature are so different! How can they go together?"

"Well," Ms. Gibbs said, "they both deal with numbers, and they both can have negative values. If a person owes money to somebody else, that's considered a negative amount of money.

"And there is a zero on the temperature scale – that's a very cold temperature – and when it is colder than this, the temperature is a negative number.

"People who live up north experience much colder temperatures than we do in Sarasota during the winter. Think about what happens as the temperature gets lower and lower and finally gets all the way down to zero degrees. What do the people who live up north do when it gets still colder?

Ms. Gibbs answered her own question: "They use negative numbers, starting at $^{-}1$, $^{-}2$, $^{-}3$, and so on, as far as they need to go. Near the South Pole the temperature is usually about $^{-}30$ degrees, but it can get colder than $^{-}100$ degrees."

"I don't think I want to live there!" I said.

"No, you probably don't," said Ms. Gibbs, "but many people in the northern US endure temperatures as low as 10 degrees regularly. Rain turns into snow and water turns into ice when the temperature goes below 32 degrees, but that almost never happens in Sarasota. Here the temperature rarely gets below about 50 degrees.

"Now here's an example of a negative amount of money: Many people use credit cards to buy things. Credit card companies let people buy things even when they don't have enough money to pay for them. And if people don't pay for everything they charged – that is, if they have a negative amount of money - at the end of the month, the credit card company charges them an extra amount of money. They end up owing more money than they actually spent."

"That doesn't seem fair," I said. "How can people ever catch up if they have to pay *extra* each month?"

"It may sound unfair," Ms. Gibbs replied, "but it's not illegal. The credit card company doesn't force anyone to charge things when they don't have enough money to pay for them. But many people may need something right away, or are too impatient to save up their money before buying something they really want. Credit card debt is widespread in our society and that's why people should understand about negative money."

"I never heard of negative money before," I said, "can you give me an example?"

"Certainly, Kayla. If someone has $**10** and wants to buy a sweater for $**15**, she might borrow $**5** from a friend. After she buys the sweater, she *owes* her friend $**5**; that's a negative amount of money. It's $^{-}\$5$.

"But people don't usually borrow money from a friend when they don't have enough to pay for what they want. They use a credit card, which is the same thing as borrowing money from the credit card company. The trouble is, if you don't pay everything you owe at the end of the month, the company charges you interest. You might end up paying much more than the cost of what you bought."

"But why don't people just save up enough money before they buy something?" I asked. "Then they wouldn't have to pay extra money to credit card companies."

“That’s easier said than done, Kayla,” said Ms. Gibbs. “Some people may have to buy food. And nowadays there’s a lot of social pressure to have ‘the latest’ jewelry or clothing, or especially a car, and some people just can’t seem to wait until they have enough money to get it.

“There is one thing, however, that almost everyone has to borrow to accomplish, and that is buying a house. But people don’t use credit cards to buy houses. They go to a bank and take out a special loan, called a mortgage, that they pay back over a period of many years. This kind of borrowing is actually a good idea, because the value of a house increases over the years and by the time it’s paid for it’s almost always worth more than the original price.

“Speaking of banks, most people have checking accounts at a bank or credit union. Writing a check is an alternative way of paying for something, usually a larger amount of money, like a monthly rent check.”

“That must be what my momma does!” I said. “I sometimes see her writing in a little book and then tearing out the page she wrote on.”

“Your mother probably has a checking account at a bank, Kayla,” Ms. Gibbs explained. “Many people put their paychecks into their checking accounts and write checks to pay the rent and monthly bills, like electricity.

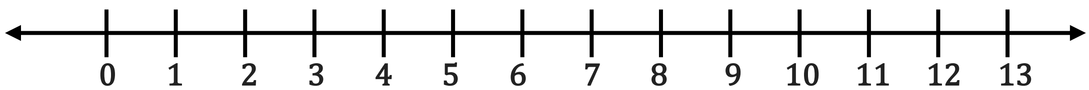
“Your mother’s bank provides a statement for her each month, which is a record of the money she put into her account and checks she wrote that took money out of her account. ‘Money in’ is called *deposits* and they have *positive* numbers in the statement. ‘Money out’ is called *debits* and they have *negative* numbers in the statement.

“Let’s use a bank statement as an example of dealing with negative numbers. Suppose you have $**50** in your checking account. Then you put another $**10** into the account, and you write a check for $**15**. The statement would contain a positive $**10** for the deposit and a negative $**15** for the debit. If we want to find the result of just these two transactions we have to subtract $**15** from $**10.**

“Huh? How can we subtract a bigger number from a smaller number? I didn’t think that was possible!”

“You’re in good company there, Kayla. Some early mathematicians didn’t know what to do with problems like this, so they just pretended they didn’t exist! But they eventually learned how to deal with them, and that’s what we’re going to talk about next: number lines.

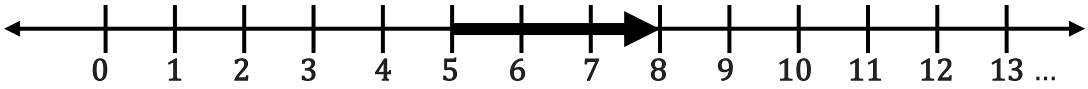
“A number line will help you understand about negative numbers.” Ms. Gibbs looked in her folder and took out a piece of paper.



“We call this a number line. As you can see, it’s a very simple idea, but it gives us a picture of the relationships among the numbers.

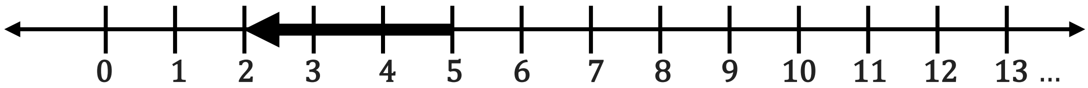
“For example, we can visualize addition and subtraction using the number line. Addition corresponds to moving to the right and subtraction corresponds to moving to the left.

“If we have the number, **5**, and we add **3** to it, we start at the **5** on the number line, and move three places to the *right*:



“You can see we end up at 8, which you know is the correct answer.

“If we have the number, 5, and we subtract 3 from it, we start at the 5 on the number line, and move three places to the *left*:



“Here the correct answer is 2, and that’s where we end up on the number line.

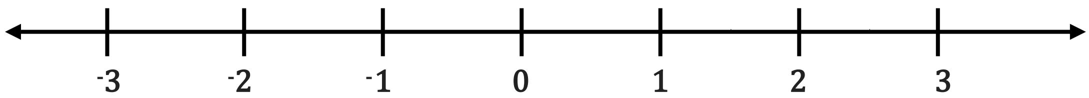
“Now let’s think about what we might do if we started at 5 and subtracted 7. This particular number line doesn’t help us here. We need to move seven places to the left, and there are only five on this number line.

“Kayla, you may have noticed the little arrows at the ends of the number line. They are meant to indicate that the line extends forever in both directions.”

“Oh-oh,” I thought to myself, “here comes ‘forever’ again, just like in repeating decimals! Well, that made sense after Ms. Gibbs showed me what it meant, so maybe lines that go on forever makes sense, too.”

Ms. Gibbs continued, “We simply renumber the line in a way that fits our problem. The big question is, what do we do when we get to the left of zero?

"As I told you at the start, negative numbers are less than zero. The first one to the left of **0** is $^{-}$**1**, the next one is $^{-}$**2**, the next one is $^{-}$**3**, and so on – forever! Remember, that's what the little arrows tell us. We never really get close to forever, but we could go as far as we need to. Here is a number line that extends to the left of zero:



"Kayla, I want to be sure that you notice where we write the negative sign in front of a number. It lies at the top left of the number.

"The reason we do that is because a negative sign in math actually has two meanings: one tells us to *do* something: subtract the following number. The other just tells us that the number lies to the *left* of zero on the number line.

"Because there are two meanings, we write the negative sign differently for each one. For a subtraction, we write it in the usual place, before the number we are subtracting:

$$7 - 5 = 2$$

"We would read this, 'seven minus five,' or some might say, 'Seven take away five.' They mean the same thing. We do NOT say, 'seven negative five.' That would be wrong because 'minus' and 'negative' have different meanings.

"For a negative number, we write the negative sign at the top left side of the number:

$$^{-}3$$

“Not everyone makes this distinction, but I think it helps our understanding if we distinguish the *property*, ‘negative’ from the *action*, ‘subtract.’ If we write negative three as

-3

we have a difficulty when we want to subtract a negative three, which I’ll tell you about when we do an example of subtracting negative numbers.

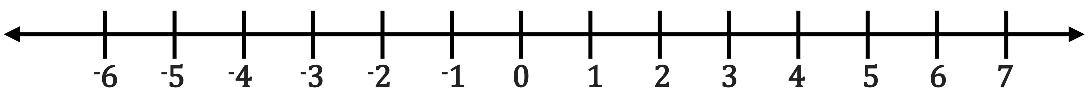
“Since most of the time we deal with positive numbers, we don’t usually include a small positive sign at the top left of a number. Sometimes it is included just for clarity’s sake, but if there is no small negative sign at the top left of a number, we assume it is a positive number.

“Sometimes it useful to think of the negative sign as meaning ‘the opposite.’ For example, the opposite of **3** is just $^{-}$**3**. It lies the same distance from zero as **3**, but on the *opposite* side of zero.

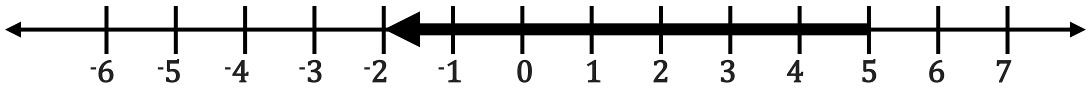
“The same reasoning tells us the opposite of $^{-}$**3** must be **3**, because it lies the same distance from zero as $^{-}$**3**, but on the *opposite* side of zero.

“Now let’s get back to using a number line to solve the problem **5** minus **7.**

“Here’s a number line with different numbers on it so that we can visualize what **5** minus **7** means:



“We start at **5** again, but this time we *subtract* **7**, which means we move **7** places to the *left*:



"You can see that we end up at negative two, $^{-}$**2**, which is the correct answer.

"Ms. Gibbs," I said, "That number line makes everything very clear. I can see now that when we subtract $**15** from $**10** we start at **10** and go **15** places to the left. That takes us to **5** places to the left of zero, or negative five, so the answer must be $^{-}$$**5**! So the balance in the checking account of the person who deposited $**10** and wrote a check for $**15** changed by $^{-}$$**5.** I think I understand about negative numbers already!"

"Well, Kayla," said Ms. Gibbs, "there are some cases where you have to think a little bit before you use the number line. In the examples so far, we've subtracted *positive* numbers. Let's think about subtracting *negative* numbers.

"Suppose you wanted to subtract $^{-}$**3** from **2**:

$$2 - {}^{-}3 = ?$$

"The idea of subtracting a negative number can be confusing. Let's do some subtractions to help us understand. First, it is obvious that subtracting anything from itself leaves zero:

$$2 - 2 = 0$$

$$^{-}2 - {}^{-}2 = 0$$

"Two minus two equals zero, and negative two minus negative two equals zero. Think about this second equation. We get the same correct result if we *add a positive two* instead of *subtracting a negative two.* It turns out that this is true in general:

Subtracting a negative number is the same as **adding the positive number.**

“Huh?” I asked. “That doesn’t seem right to me.”

“I know it sounds strange, Kayla, so let’s look at a real-life example:

“Suppose you owe your friend **20¢.** As we said before, that is the same as saying you have a negative amount of money: $^{-}$**20¢.** If you pay your friend back, you are *taking away* the debt. *Taking away* is the same as *subtracting*. Before you pay your friend back, you have $^{-}$**20¢.** After you pay your friend back, that is, you take away, or subtract, the $^{-}$**20¢**, you have zero cents. We would write this down like this:

$$^{-}20¢ - {}^{-}20¢ = 0¢$$

“What we’re saying is that this is exactly the same as:

$$^{-}20¢ + {}^{+}20¢ = 0¢$$

“In the first equation we *subtracted* a *negative* **20¢**; in the next equation we *added* a *positive* **20¢.** And you can see that we got the same answer both ways.”

“I see that it’s true,” I said, “but it still sounds funny to me that subtracting a negative is the same as adding a positive.”

“That’s because it is a new idea for you, Kayla,” said Ms. Gibbs. “It is always good to be skeptical about a new idea until you have convinced yourself that it’s true.

Now, suppose we subtract $^{-}$**3** from **2**:

$$2 - {}^{-}3 = ?$$ is the same as $$2 + {}^{+}3 = ?$$

"Which is **5**," I said, but I was thinking, "Hmm, things don't seem so simple anymore…" Then I said to Ms. Gibbs, "Can we do another example of subtracting a negative number?"

"Sure, Kayla. What is **3** minus negative **5**?

$$3 - {}^{-}5 = ?$$

"Well," I replied, "the negative number we're subtracting is $^{-}$**5**, so that must be the same as adding a positive **5**. We can rewrite this equation as:

$$3 + {}^{+}5 = ?$$

"And that's easy! I don't even need the number line to know that the answer is **8**."

"Good for you, Kayla," said Ms. Gibbs. "Let's try one more problem before moving on to multiplication and division with negative numbers." What is negative three minus negative eight? First write this as an equation."

"OK," I said. "I think it would be:

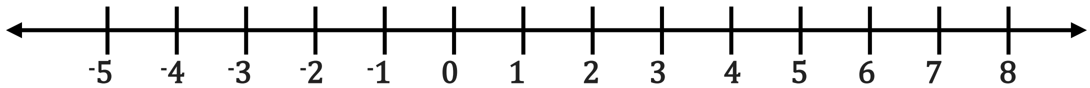
$${}^{-}3 - {}^{-}8 = ?$$

"And, just like before, subtracting a negative **8** is the same as adding a positive **8**, so I can rewrite the equation like this:

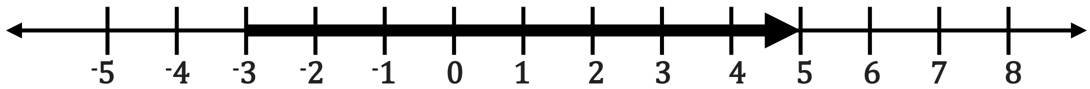
$${}^{-}3 + {}^{+}8 = ?$$

"Uh, Ms. Gibbs, I think I'd like to use the number line to solve this equation."

“That’s a very good idea, Kayla. If you’re not sure how to solve the equation, the number line helps you to see how to do it. Here is a number line for you to use”:



“OK. I start out at $^{-}3$ because that’s the number I’m adding the positive 8 to. Adding positive 8 means moving 8 places to the *right* on the number line:



Starting at $^{-}3$ and counting 8 places to the right, I end up on positive 5. My answer is 5.”

“Excellent work, Kayla! You are a fast learner.”

“Well,” I said, “You make things that sound complicated at first seem easy after you explain them and show me some examples.”

“I’m glad to hear you say that, Kayla, but you deserve a lot of credit yourself. You clearly practice doing problems, and that always makes learning new things easier.

“Another thing that makes learning new things easier is relating the new ideas to something we are familiar with. For example, you know what happens when we put an ice cube into a glass of water.”

“Yes,” I said, the water gets colder.”

“That’s right, Kayla. The temperature goes down, and the more ice you add, the colder the water gets, until the water is the same temperature as the ice.

“Here’s a pretend story that may help you understand negative numbers better:

A king in a faraway land has magic cubes that his servants use to adjust the temperature of the water he drinks. There are hot cubes to make the water warmer, and cold cubes to make the water cooler. The cubes are magic; unlike ice cubes, the cold ones don’t melt, and both the hot ones and the cold ones can be used over and over.

For each hot cube they put **into** the water, the temperature goes up **1** degree. For each hot cube they **remove** from the water, the temperature goes down **1** degree.

The cold cubes work the opposite way. For each cold cube they put **into** the water, the temperature goes down **1** degree. For each cold cube they **remove** from the water, the temperature goes up **1** degree.

“We can relate the addition or removal of these cubes to addition and subtraction of positive and negative numbers. We’ll use a little plus sign at the top left of the hot cubes and a little negative sign at the top left of the cold cubes.

“For example, if **4** hot cubes and **10** cold cubes are added to the water, how would the temperature change? The **4** hot cubes would raise the temperature by **4** degrees and the **10** cold cubes would decrease the temperature by **10** degrees.

“The equation for this can be written as:

$$^{+}4 + {}^{-}10 = {}^{-}6.$$

“So the temperature would *decrease* by **6** degrees. That means the same thing as ‘the temperature changes by negative six degrees.’

Reader, if you’re not sure why the answer is $^{-}6$ degrees, use a number line to solve the equation.

“For another example, if **3** hot cubes were added and **2** cold cubes were removed from the water, the temperature would increase by **5** degrees. The equation for this can be written as”:

$$^{+}3 - {}^{-}2 = {}^{+}5$$

“The minus sign between the $^{+}3$ and the $^{-}2$ means we are *removing* the cold cubes.”

“Oh, Ms. Gibbs,” I said, “You’re right! I can see now that taking away two cold cubes is the same as adding two hot cubes; the temperature of the water changes in the same way whichever you do.

“It seems much clearer to me than just saying ‘subtracting a negative number is the same as adding a positive number.’ That story really helps me understand this idea!”

“Let’s see how well you understand, Kayla,” said Ms. Gibbs, “Can you write the equation that represents adding five cold cubes and adding two hot cubes?”

“Well, the way to write five cold cubes is $^{-}5$ and the way to write two hot cubes is $^{+}2$. And since we’re adding both, the equation would be:

$$^{-}5 + {}^{+}2 = ?$$

“First the water gets cooled off by five degrees, and then it gets warmed up by two degrees. Those two degrees cancel out two of the five degrees the water was cooled, so now it is three degrees cooler than when we started:

$$^{-}5 + {}^{+}2 = {}^{-}3$$

“That’s right, Kayla, said Ms. Gibbs. With some practice you will have no trouble adding and subtracting negative numbers.

“There is one more use of the number line that helps us in our study of negative numbers. As we go from left to right, the numbers get larger. Any number on the line is less than all the numbers to the right of it, and greater than all the numbers to the left of it.

“And there are special symbols in mathematics for ‘less than’ and ‘greater than.’ ‘Less than’ is represented by the symbol, < , and ‘greater than,’ by the symbol, > .

“For example, we can write:

$4 < 9$ which means “four is less than nine”

and

$12 > 7$ which means “twelve is greater than seven”

“An easy way to remember which is which is to notice that each symbol has a large, open side and a small side that comes to a point. The large side is always towards the larger number and the small side is always towards the smaller number."

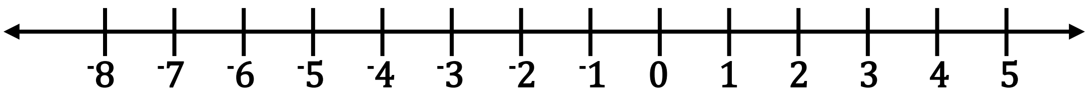
"That's easy to remember," I said, "I don't think I'll ever forget that!"

"No, I'm sure you won't, Kayla," Ms. Gibbs said. "Incidentally, we don't refer to these two expressions as 'equations.' After all, the left side never *equals* the right side! The proper expression is 'inequalities.'

"Some students are confused by the inequality:

$$^{-}\mathbf{6} < \mathbf{3}$$

"Because six is greater than three, it doesn't seem right to have the 'less than' symbol here. But if we look at these two numbers on a number line, it is clear that $^{-}\mathbf{6}$ lies to the left of **3**, and is therefore less than **3**:



We can also see that

$$^{-}\mathbf{8} < {}^{-}\mathbf{6} \quad \text{and} \quad {}^{-}\mathbf{4} > {}^{-}\mathbf{7} \quad \text{and} \quad \mathbf{0} > {}^{-}\mathbf{1}$$

"All you have to remember is that any number on the line is greater than numbers to the left of it, and less than those to the right of it, and you can't go wrong.

"Before going on, Kayla," said Ms. Gibbs, "I want to show you the way to write an equation for subtracting a negative number if you don't use the small negative sign at the top left of the number. Remember when I told you that leads to a difficulty?

“Suppose we want to write the equation for subtracting a negative five from positive three. We would simply write:

$$3 - {}^{-}5 = ?$$

“But if we were to write negative five as -5, the equation looks like this:

$$3 - -5 = ?$$

“Because this might be confusing, parentheses are placed around the negative five so that the equation is written:

$$3 - (-5) = ?$$

“You should know both ways of writing such an equation because even though using the way I showed you first is better, not everyone uses it.

“Finally, I think we’re ready to move on to multiplying and dividing with negative numbers.

“There are two simple rules that apply to both multiplication and division with negative numbers:

1. A positive and a negative give a negative answer.

2. A negative and a negative give a positive answer.

“Huh?” I asked.

“Let’s do some multiplication examples first:

$$3 \times {}^{-}2 = {}^{-}6$$

"Here we have one positive and one negative, so the answer is negative. This is easily understood if we think of multiplication as repeat addition. Three times negative two means negative two plus negative two plus negative two, and that is negative six:

$$^{-}2 + {}^{-}2 + {}^{-}2 = {}^{-}6$$

"We would get the same answer for $^{-}2$ x 3, 2 x $^{-}3$, and $^{-}3$ x 2, because all of these have one positive and one negative.

"Here's an example with two negatives:

$$^{-}3 \times {}^{-}5 = {}^{+}15$$

"I put a little positive sign on the **15** just to emphasize that the two negatives multiplied together give a positive answer. This rule is not so easy to understand, so you'll just have to accept it until you get to more advanced math topics.

"Now here's a multiplication with three digits. You tell me what the answer is:

$$^{-}4 \times 3 \times {}^{-}2 = ?$$

"Well," I said, "I think I can just do one multiplication at a time. First, negative four times positive three is negative twelve, because it is one negative times one positive:

$$^{-}4 \times 3 = {}^{-}12$$

"And now I have:

$$^{-}12 \times {}^{-}2 = {}^{+}24$$

"The answer, **24**, is positive because we have a negative times a negative."

“Very good, Kayla! Let’s try one more. What is:

$$^{-}3 \times {}^{-}2 \times {}^{-}1 = ?$$

“I can do one multiplication at a time again,” I said. “Negative three times negative two is positive six:

$$^{-}3 \times {}^{-}2 = {}^{+}6$$

“and positive six times negative one is just negative six:

$$^{+}6 \times {}^{-}1 = {}^{-}6$$

“So my answer is negative six.”

“Good work, Kayla,” said Ms. Gibbs. “Now division with negative numbers should be just as easy for you. Let’s start with:

$$12 \div {}^{-}3 = ?$$

“Well, I know that twelve divided by three is just 4, but because there is one negative and one positive, the answer must be negative:

$$12 \div {}^{-}3 = {}^{-}4$$

“The answer is negative four.”

“That’s right, Kayla,” said Ms. Gibbs. Now sometimes you see division problems written this way, but the rules apply just the same:

$$\frac{12}{^{-}3} = {}^{-}4$$

“I think one more example is all you need on this topic, Kayla. Here’s a problem that combines multiplication and division, but if you follow the rules as you have been doing, I’m sure you’ll get the correct answer”:

$$\frac{{}^-5 \text{ x } 6}{{}^-2} = ?$$

“I think I’ll do the numerator first,” I said. “Negative five times positive six is negative thirty. Now the problem becomes:

$$\frac{{}^-30}{{}^-2} = ?$$

“And I know how to do short division:

$$\begin{array}{r} 1\ 5 \\ 2\overline{)\ 3^{1}0} \end{array}$$

“So two goes into thirty fifteen times. And because there are two negatives, the answer must be positive:

$$\frac{{}^-30}{{}^-2} = {}^+15$$

“I knew you would be able to solve that problem, Kayla!” said Ms. Gibbs. “You see that it’s just a matter of following those two simple rules and doing one step at a time.”

Finally Ms. Gibbs was all done with those negative numbers. Now I can tell her about me making a basket and that I made that basket in a real game. But before I got a chance, Ms. Gibbs said she had something very important to tell me.

Ms. Gibbs said, very serious-like, “Kayla, next Thursday will be the very last day I’ll be tutoring you for this school year so I thought we’d do something special.

I began to smile because I thought she would say something like - “so let’s have a little party, that would be special,” - But she didn’t say that! I’ll tell you what she said”:

“I thought next week would be a good day for you to do some problem-solving.” Ms. Gibbs said it with a smile.

Chapter 4

Reviewing what I learned

Today I learned about negative numbers. Negative numbers are just like regular numbers but they have a negative sign in front of them, like this: $^{-}$**5**.

All the numbers we talked about so far are positive even though we didn't call them that. You see, positive numbers don't have a sign in front of them telling you they're positive. You're just supposed to know that!

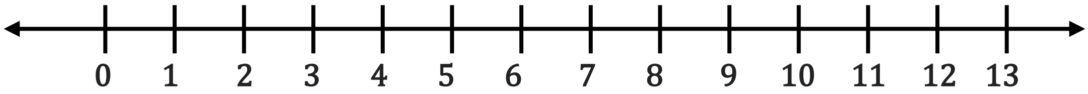
But negative numbers have a little minus sign in front of them to tell you they're negative. We use negative numbers in our everyday lives, so it's a good idea to understand what it means for a number to be negative. That's what Ms. Gibbs says.

Now you may be wondering when you use a negative number. Well, for one thing, negative numbers are used to describe the temperature in the north because sometimes, when it's really cold, the temperature gets below zero. Really it does! Maybe you live in the north and know about temperatures that are below zero. Well, if you do, then you already know a little about negative numbers.

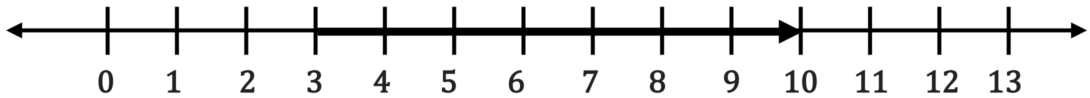
If you don't live in the north, I'll explain. Think about what happens as the temperature gets lower and lower and finally gets all the way down to zero degrees. What do they do when it gets still colder? Well, they have to use negative numbers! That's what!

If the temperature goes below zero, say **10** degrees below zero, you have to say it's minus **10** degrees outside (and write $^{-}$**10**). That's really cold! And it can get even colder than that. In some places it can get **20** or **30** degrees below zero ($^{-}$**20** or $^{-}$**30**) or even more. If you go outside when it's that cold, make sure you wear mittens or gloves or else your fingers will freeze!

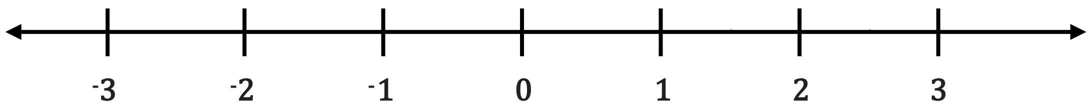
When you're adding or subtracting negative numbers, it can be tricky so it's good to use a number line. Ms. Gibbs gave me one and here it is:



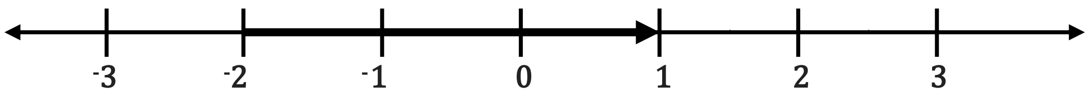
"Now suppose we have the number **3**, and we add **7** to it, we start at the **3** on the number line, and move **7** places to the *right*:



And we get **10**. You knew that already, I bet. That's just adding regular numbers. But wait, here's a number line that has negative numbers on it:



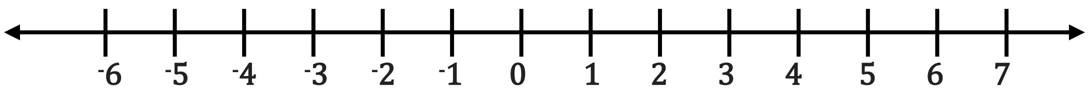
If we start at $^{-}$**2** and add $^{+}$**3**, our answer is only $^{+}$**1**. Really it is. Just use the number line and you'll see. You start at $^{-}$**2** and you move three spaces to the right, that's because you're adding $^{+}$**3**, and you end up with a positive one. Try it yourself:



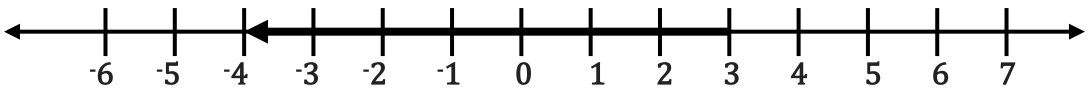
So when we *add,* we move to the right, when we *subtract,* we move to the left. It's as simple as that!

Number lines are really helpful when you want to add numbers with a negative sign. If you don't use them at first, you might get mixed up and get the wrong answer. When I first started with negative numbers, I used them all the time. Now that I'm just finishing high school, I don't use them at all, not really! But I'm glad I learned how to use them.

Suppose I want to subtract **7** from **3**. You might say something like, "You can't do that!" Well, you can, really! I'll show you and I'll use the number line so you can see what I'm doing:



I start on the number I'm going to subtract from, three, and go left (because I'm subtracting) seven places:



See, I did it! And I got $^{-}\mathbf{4}$ and you'll get $^{-}\mathbf{4}$ too, that is, if you do it right!

Oh, there is another thing I learned today. There are two new signs that you have to know about: > and <. The first sign means "greater than" and the second sign means "less than."

I used to get these two signs mixed up until I figured out a trick to help me remember. It's not a trick, really, it's a mnemonic device. I'll tell you what it is: When the *skinny* part of the sign is first, you read it as "less than." Get it? "Less than" is *skinnier*.

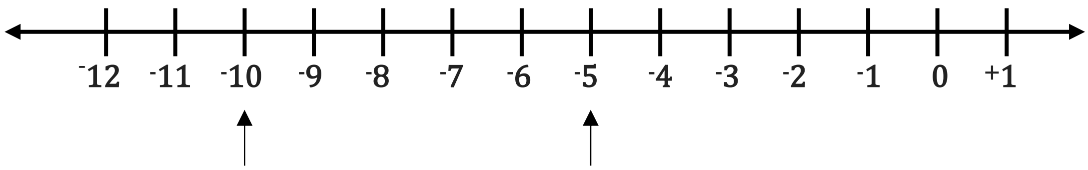
The sign for "greater than" is this: >. You see, the *fat* part of the sign is first, and you read it as "greater than." Do you get it? "Greater than" is *fatter*.

So if the fat part goes first, like this: >, it reads “greater than.” If the skinny part goes first, like this <, it reads “less than.”

This is easy to understand as long as we have only positive numbers. It’s pretty clear that $2 < 3$ (“2 is less than 3”) and $9 > 5$ (“9 is greater than 5”), but when we have negative numbers it can be confusing unless we use a number line.

Suppose we want to know which is greater, $^{-}5$ or $^{-}10$. Our first reaction might be that $^{-}10$ is greater than $^{-}5$, because 10 is greater than 5. But we just have to remember that on the number line numbers always get greater as we go from left to right.

Let’s look at a number line with these two numbers on it:



$^{-}5$ lies to the right of $^{-}10$, so it must be greater:

$$^{-}5 > {}^{-}10$$

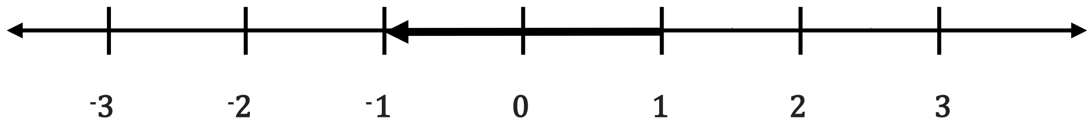
We could also say that $^{-}10$ is less than $^{-}5$:

$$^{-}10 < {}^{-}5$$

Oh, and there is another new word for you to learn: “inequality.” These mathematical statements are *not* equations! They’re “inequalities.” And we can say that these last two inequalities mean the same thing.

I told you about negative temperatures already, but there is another very important time you use negative numbers: with money. You see, some people buy things that they don’t have enough money to pay for. When they do that, they have a *negative* amount of money. I'll give you an example:

Just suppose you want to buy a book and it costs $2 but you only have $1. Well you can wait until you have enough money - or you can borrow money from someone, say your mom - just make sure you pay her back. Now you have a negative $1, which we can write as ⁻$1. If you use a number line, you can see it. Here I'll show you:



You start with $1, that's a positive one on the number line. Then you spend $2. That means you have to *subtract* 2, because when you spend $2 you have $2 *less* than you did before you spent it. To subtract 2 on the number line we move two places to the left. You can see that we end up on -1, which means you *owe* your mom the $1 she lent you to buy the book.

Negative money is money you owe someone. Having negative money is worse than having NO money!

Most grown-up don't borrow from their moms though, they borrow from a bank. Banks have a lot of money and they can make even more money by lending the money they have to people. Say you want to buy something really, really big, like a house. Very few people have the money to buy a house so they go to a bank and take out a mortgage. A mortgage is just a fancy word for a loan for something big.

It's a good idea to borrow money for a house, but it's *not* a good idea to borrow money for some clothes. It's better to save up for clothes because of the extra interest you have to pay when you borrow the money. But borrowing money for a mortgage on a house is OK because very few people have all that money in the bank. And besides, you don't have to pay any rent! Really you don't!

Negative numbers are used for the temperature and for money so you have to learn about them. I can't think of anything else negative numbers are used for, can you?

Chapter 5

About problem-solving

Well, Kayla, you certainly have come a long way since we started our meetings. We have covered many topics in math and you have worked hard to gain a very sound understanding of the basic principles. I think maybe it's time we looked into the idea of "problem-solving."

"Problem-solving?" I asked. "Why Ms. Gibbs, I've been working problems and solving them all year…haven't I?"

"Well, those were *number* problems," Ms. Gibbs said. "The kind of problems I'm talking about now are *word* problems. With number problems you know what to do with the numbers; you're told to add, subtract, multiply or divide.

"But you see, word problems don't tell you *what* to do, you have to figure it out yourself. And it's good that you've already done a lot of number problems because after you figure out what to do when problem-solving, you frequently have to do the math. With word problems, though, you have to think first before you do anything.

"I have to *think*?" I said. "But Ms. Gibbs, I always have to think when I'm doing my math."

But Ms. Gibbs didn't pay attention to me saying that, even though I said it loudly. She just continued talking about those kinds of problems - you know - those word problems. Ms. Gibbs explained, "You'll have to first *think* how you would go about solving a problem before you even begin to do the work.

“Let me give you an example. An ice cream cone and a piece of candy together cost $1.10, and the ice cream cone costs $1 more than the piece of candy. How much does each one cost?”

“That’s easy,” I said, “The ice cream cone costs $1 and the candy costs 10¢.”

“Ah, you see, Kayla?” said Ms. Gibbs. “You answered without really thinking. It sounds like a very easy problem, but it’s not. Think about it. If the candy costs 10¢ and the ice cream cone costs $1 more than that, the ice cream cone would cost $1.10. Together they would cost $1.20, so your answer can’t be right.”

I was really confused. I didn’t know what to say. Ms. Gibbs must have seen that I was embarrassed, because she said, “Don’t feel bad, Kayla. Most adults I’ve given this problem to gave the same answer you did. I know that if you think about it for a while, you’ll get the right answer.

“I’m not going to help you get the answer because developing problem-solving skills requires some struggling! The more you think about a problem on your own, the more you develop problem-solving skills. When I help you, I take away your opportunity to think about the problem yourself. That slows down your progress in learning how to solve problems.

“You work on this problem on your own this week, and we’ll talk more about it at our next meeting.

Practice problems

1. Perform the following operations:

a) $4 + {}^{-}2 =$	b) ${}^{-}7 \times 3 =$	c) ${}^{-}10 \div 5 =$
d) $10 - {}^{-}12 =$	e) ${}^{-}6 \div {}^{-}2 =$	f) ${}^{-}7 \times {}^{-}4 =$
g) ${}^{-}9 + {}^{-}6 =$	h) $0 \times {}^{-}8 =$	i) $0 + {}^{-}3 =$

2. In Death Valley, CA, the average high temperature in July is 116 degrees. In Deadhorse, AK, the average low temperature in February is ${}^{-}23$ degrees. What is the difference between these two temperatures?

__________degrees

3. On a winter Sunday up north, the temperature was 5 degrees. On Monday the temperature had dropped by 12 degrees. What was the temperature on Monday?

__________degrees

4. In Fairbanks, AK, the temperature dropped by 5 degrees in one hour and then the temperature was ${}^{-}9$ degrees. What was the temperature at the beginning of that hour?

__________degrees

5. A fisherman wanted to put his bait 2.5 meters off the bottom of the lake. If the water where he's fishing is 10 meters deep, how deep will the bait be? (Remember, the positive direction is UP and the negative direction is DOWN.)

________meters

6. On a certain test, each student got 5 points for every correct answer, but negative 3 points for every wrong answer.

a) One student had 15 correct answers and 5 wrong answers. What was this student's score on this test?

Score=______

b) Another student got only 6 correct answers and 12 wrong answers. What was this student's score on this test?

Score=______

7. Bella was standing on a hill that was 25 meters higher than her house. There was another hill that she wanted to get to the top of. To do that she would have to go down 15 meters, then back up 30 meters. How high above Bella's house was the second hill? (It might help to draw a picture.)

________meters

8. When keeping financial records, positive numbers are used for money you *have*, and negative numbers are used for money you *owe* to somebody else. A business or a person has a “Net Worth” equal to the sum of these numbers.

a) Mr. Smith has $1,235 in a savings account, $567 in a checking account, and he owes a credit card company $750. What is his Net Worth?

_________dollars

b) The credit card company charges 18% interest per year if Mr. Smith doesn’t pay the $750. How much will Mr. Smith owe the credit card company one year from now if he doesn’t make any payments?

________dollars

c) If Mr. Smith puts $200 more into his savings account during the year and his checking account is less by $75 after the year, what is his Net Worth after the year?

________dollars

Here are some review problems to reinforce what you learned in previous lessons:

Perform the following operations with fractions:

9a) $\frac{1}{3} + \frac{2}{36} =$ 9b) $\frac{2}{5} \times \frac{2}{4} =$ 9c) $\frac{4}{7} - \frac{5}{35} =$ 9d) $\frac{3}{5} \div \frac{9}{20} =$

9e) $2\frac{2}{5} + 3\frac{2}{10}$ 9f) $2\frac{2}{5} \times 3\frac{2}{10} =$ 9g) $2\frac{2}{5} \div 3\frac{2}{10} =$

Perform these short division problems:

10a) $2\overline{)\,1\ 5\ 4}$ 10b) $5\overline{)\,9\ 7\ 0}$ 10c) $3\overline{)\,1\ 6\ 2}$ 10d) $7\overline{)\,2\ 5\ 3\ 4}$

Convert the following fractions to decimals:

11a) $\frac{2}{5}$ 11b) $\frac{3}{8}$ 11c) $\frac{3}{4}$ 11d) $\frac{5}{6}$

Write the decimals in problem 11 as per cents:

11a) 11b) 11c) 11d)

Write the following decimals as fractions (simplify to lowest terms):

12a) 0.5 12b) 0.0035 12c) 0.025 12d) 0.120

Something extra

I decided to include a little Spanish in the rest of the Kayla books, but just a little bit - after the math lesson, of course. You see, I'm learning more and more Spanish every day and I want to share some with you.

Today for my "Something Extra" I'm going to talk about punctuation. You know, like question marks and exclamation points. Let's take a question mark for now. We have these in English too but for people at the end of the question. And you'll never guess about the one in the beginning. Ready for it? it's upside down! I'm not making that up. It's true! It's like this: ¿ If someone writes, "How are you?" in Spanish, it looks like this: "¿Como estas?"

I thought of why they do that and I think I figured it out. You see, in English, you don't know until you're all done reading a question that what you're reading is a question. But in Spanish, right when you start reading the question, you know. That's pretty neat.

I bet kids who speak Spanish and are just learning English think it's pretty funny that we don't have our question marks at the beginning of the sentence and that ours are always right side up!

And it's the same thing for exclamation points. There are two of them and the first one, the one at the beginning of the sentence, is up-side-down too.

I have more tricky things to tell you - like about nouns having gender - but I'll tell you next time. I hope you want to learn Spanish someday soon.

Answers

1a) $^{+}2$ or just 2   1b) $^{-}21$ 1c) $^{-}2$

1d) 22   1e) 3   1f) 28

1g) $^{-}15$ 1h) 0   1i) $^{-}3$

2. 116 degree - $^{-}23$ degrees = 139 degrees

3. 5 degrees – 12 degrees = $^{-}7$ degrees

4. Represent the unknown temperature with a question mark:

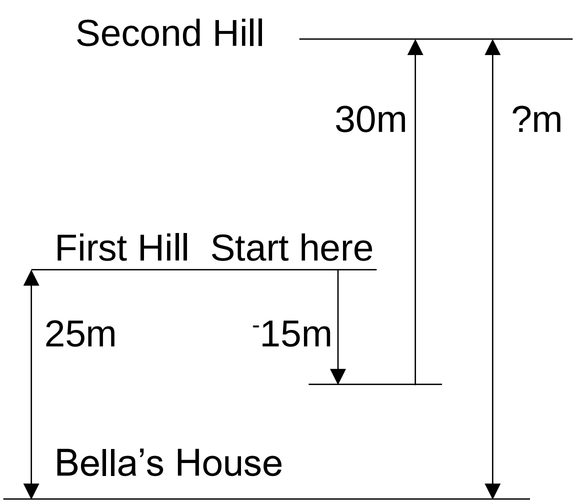
$$? - 5 = {}^{-}9; \quad ? = {}^{-}4$$

5. $^{-}10$ meters + 2.5 meters = $^{-}7.5$ meters

6a) $15 \times 5 + 5 \times {}^{-}3 = 75 + {}^{-}15 = 60$

6b) $6 \times 5 + 12 \times {}^{-}3 = 30 + {}^{-}36 = {}^{-}6$

7.



We start on top of the first hill, that's at 25m. Then we go down 15m and up 30m:

$$25m + {}^{-}15m + 30m = 40m$$

8a) $1,235 + $567 + ⁻$750 = $1052

8b) Remember, to get a percent of a number we multiply by the percent and move the decimal point two places to the left:

$$\begin{array}{r} \$750 \\ \underline{\times 18\%} \\ 6000 \\ \underline{750} \\ \$13500 \end{array} \longrightarrow \$135.00$$

This is just the interest. It must be added to the amount already owed:

$$\begin{array}{r} \$750 \\ \underline{\$135} \\ \$885 \end{array}$$

8c) $1235 + $200 + $567 - $75 + ⁻$885 = $1042

Mr. Smith's Net Worth has gone down during the year, mainly because he didn't pay off his credit card debt.

Review problems

9a) $\frac{14}{36} = \frac{7}{18}$ 9b) $\frac{4}{20} = \frac{1}{5}$ 9c) $\frac{15}{35} = \frac{3}{7}$ 9d) $\frac{60}{45} = \frac{4}{3}$

9e) $\frac{12}{5} + \frac{32}{10} = \frac{56}{10} = \frac{28}{5}$ 9f) $\frac{12}{5} \times \frac{32}{10} = \frac{384}{50} = \frac{192}{25}$

9g) $\frac{12}{5} \times \frac{10}{32} = \frac{120}{160} = \frac{3}{4}$

10a) $\begin{array}{r} 7\,7 \\ 2\overline{)\,1\,5^{1}4} \end{array}$ 10b) $\begin{array}{r} 1\;9\,4 \\ 5\overline{)\,9^{4}7^{2}0} \end{array}$ 10c) $\begin{array}{r} 5\,4 \\ 3\overline{)\,1\,6^{1}2} \end{array}$ 10d) $\begin{array}{r} 3\,6\;2 \\ 7\overline{)\,2\,5^{4}3^{1}4} \end{array}$

11a) 0.4 11b) 0.375 11c) 0.75 11d) $0.8\overline{3}$

11a) 40% 11b) 37.5% 11c) 75% 11d) $83.\overline{3}\%$

12a) $\frac{5}{10} = \frac{1}{2}$ 12b) $\frac{35}{10{,}000} = \frac{7}{2000}$

12c) $\frac{25}{1000} = \frac{1}{40}$ 12d) $\frac{120}{1000} = \frac{3}{25}$

And, finally, the grammar mistake Jake made when he said, “I’m not playing with no girls!”: This sentence is an example of a “double negative” (“not” and “no” in the same sentence) which is not considered acceptable usage. He should have said, “I’m not playing with *any* girls!”

About tutoring

Negative numbers can be very confusing. That's why Ms. Gibbs used a number line to help Kayla understand.

When you're tutoring, find ways to help the person you're tutoring understand what you're trying to explain. If the person lives in the north where it gets quite cold, they are probably familiar with negative numbers. But if the person you're tutoring lives in the south, they might have difficulty imagining negative numbers, at least for the weather.

But everyone needs to understand negative numbers when it comes to money. Credit card debt is a widespread problem in the US. If we can convince youngsters that spending money that they don't have is a very serious mistake, that would be one of the most important lessons they learn from their tutors.